ventricle, and the extravaſation covering the fiſſure in the aorta, exactly marked, as they appeared to,

My Lord,

Your Lordſhip's

moſt obedient

and moſt humble ſervant,

Frank Nicholls.

LII. *Of the Irregularities in the planetary Motions, cauſed by the mutual Attraction of the Planets: In a Letter to* Charles Morton, *M. D. Secretary to the Royal Society, by* Charles Walmeſley, *F. R. S. and Member of the Royal Academy of Sciences at* Berlin, *and of the Inſtitute at* Bologna.

S I R,

Read Dec. 10, 1761.

FInding that the influence, which the primary planets have upon one another, to diſturb mutually their motions, had been but little conſidered, I thought it a ſubject worthy of examination. The force of the ſun, to diſturb the moon's motion, flows from the general principle of *gravitation*, and has been fully aſcertained, both by theory and obſervation; and it follows, from the

ſame

ſame principle, that all the planets muſt act upon one another, proportionally to the quantities of matter contained in their bulk, and inverſe ratio of the ſquares of their mutual diſtances; but as the quantity of matter contained in each of them, is but ſmall when compared to that of the ſun, ſo their action upon one another, is not ſo ſenſible as that of the ſun upon the moon. Aſtronomers generally contented themſelves with ſolely conſidering thoſe inequalities of the planetary motions, that ariſe from the elliptical figure of their orbits; but as they have been enabled, of late years, by the perfection of their inſtruments, to make obſervations with much more accuracy than before, they have diſcovered other variations, which they have not, indeed, been able yet to ſettle, but which ſeem to be owing to no other cauſe, but the mutual attraction of thoſe celeſtial bodies. In order, therefore, to aſſiſt the aſtronomers in diſtinguiſhing and fixing theſe variations, I ſhall endeavour to calculate their quantity, from the general law of gravitation, and reduce the reſult into tables, that may be conſulted, whenever obſervations are made.

I offer to you, at preſent, the firſt part of ſuch a theory, in which I have chiefly conſidered the effects produced by the actions of the earth and Venus upon each other. But the ſame propoſitions will likewiſe give, by proper ſubſtitutions, the effects of the other planets upon theſe two, or of theſe two upon the others. To obviate, in part, the difficulty of ſuch intricate calculations, I have ſuppoſed the orbits of the earth and Venus to be originally circular, and to ſuffer no other alteration, but what is occaſioned by their mutual attraction, and the attraction of the other

planets.

planets. Where the forces of two planets are confiderable, with refpect to each other, as in the cafe of Jupiter and Saturn, it may be neceffary, in fuch computations, to have regard to the excentricity of their orbits; and this may be referved for a fubject of future fcrutiny. But the fuppofing the orbits of the earth and Venus to be circular, may, in the prefent cafe, be admitted, without difficulty, as the forces of thefe two planets are fo fmall, and the excentricity of their orbits not confiderable. On thefe grounds, therefore, I have computed the variations, which are the effects of the earth's action: firft, the variation of Venus's diftance from the fun; fecondly, that of its place in the ecliptic; thirdly, the retrograde motion of Venus's nodes; and, fourthly, the variation of inclination of its orbit to the plane of the ecliptic.

The fimilar irregularities in the motion of the earth, occafioned by its gravitation to Venus, are here likewife computed: but it is to be obferved, that the abfolute quantity of thefe irregularities is not here given, it being impoffible, at prefent, to do it; becaufe the abfolute force of Venus is not known to us. I have, therefore, ftated that planet's force by fuppofition, and have, accordingly, computed the effects it muft produce; with the view, that the aftronomers may compare their obfervations with the motions fo calculated, and, from thence, difcover how much the real force differs from that which has been fuppofed. But the exact determination of the force of Venus muft be obtained, by obfervations made on the fun's place, at fuch times, when the effect of the other planets is either null or known.

The influence of Venus upon the earth being thus computed, that of the other planets upon the ſame, may likewiſe, hereafter, be conſidered: by which means, the different equations, that are to enter into the ſettling of the ſun's apparent place, will be determined; the change of the poſition of the plane of the earth's orbit will alſo be known; and, conſequently, the alteration that thence ariſes in the obliquity of the ecliptic, and in the longitude and latitude of the fixed ſtars. Theſe matters of ſpeculation are reſerved for another occaſion, in caſe what is here offered ſhould deſerve approbation.

I am glad to have it in my power to preſent you with this teſtimony of my gratitude for paſt favours, and of my reſpect for your diſtinguiſhed merit; and it is with ſincerity, I ſubſcribe myſelf,

SIR,

Your very humble ſervant,

Bath,
Nov. 21, 1761.

Cha. Walmeſley.

De Inæqualitatibus quas in motibus Planetarum generant ipſorum in ſe invicem actiones.

QUoniam in theoriæ hujus decurſu frequens erit uſus fluentium quæ arcubus circuli, vel eorum ſinibus, coſinibus, et ſinibus verſis, exprimuntur, idcircò lemma ſequens, quod alibi olim tradidi, lubet hîc apponere.

LEMMA.

LEMMA.

Dato cosinu arcûs cujusvis, invenire cosinum et sinum arcûs alterius qui sit ad priorem in ratione λ ad 1.

Detur c cosinus arcûs A ad radium 1, et sit arcus $B = \lambda A$, cujus cosinus dicatur t; eritque, ut notum est, $\dot{A} = \frac{-\dot{c}}{\sqrt{1-cc}}$, atque $\dot{B} = \lambda\dot{A} = \frac{-\dot{t}}{\sqrt{1-tt}}$. Ponatur $c = \frac{1+xx}{2x}$, et $t = \frac{1+yy}{2y}$, fietque $\dot{A} = \frac{\dot{x}}{x\sqrt{-1}}$, $\dot{B} = \frac{\dot{y}}{y\sqrt{-1}}$: sed est $\dot{A} . \dot{B} :: 1 . \lambda$, adeoque $\frac{\lambda\dot{x}}{x} = \frac{\dot{y}}{y}$; unde log. $x^{\lambda} =$ log. y, et $x^{\lambda} = y$. Verùm æquationes $c = \frac{1+xx}{2x}$ et $t = \frac{1+yy}{2y}$ dant $x = c + \sqrt{cc-1}$, $x = c - \sqrt{cc-1}$, et $y = t + \sqrt{tt-1}$, $y = t - \sqrt{tt-1}$; unde est $x^{\lambda} = t \pm \sqrt{tt-1} = \overline{c \pm \sqrt{cc-1}}^{\lambda}$, atque inde $2t = \overline{c + \sqrt{cc-1}}^{\lambda} + \overline{c - \sqrt{cc-1}}^{\lambda}$. Fiat igitur $c + \sqrt{cc-1} = l$, et $c - \sqrt{cc-1} = m$, eritque $lm = 1$, et $c =$ cos. $A = \frac{l+m}{2}$, et sin. $A = \frac{l-m}{2}\sqrt{-1}$; atque inde $t =$ cos. $B = \frac{l^{\lambda}+m^{\lambda}}{2}$, et sin. $B = \frac{l^{\lambda}-m^{\lambda}}{2}\sqrt{-1}$.

Itaque in circulo, cujus radius est 1, si duorum arcuum vel angulorum A et B alteruter B sit ad alterum A ut numerus quilibet λ ad 1, et ponatur cos. $A = \frac{l+m}{2}$, existente $lm = 1$, erit sin. A

$=$

$= \frac{l-m}{2}\sqrt{-1}$, atque cos. B = coss λA $= \frac{l^{\lambda}+m^{\lambda}}{2}$, et sin. B = sin. λA $= \frac{l^{\lambda}-m^{\lambda}}{2}\sqrt{-1}$. Q. E. I.

COROLL. I.

Hinc habetur cos. A $\times$ cos. B $= \frac{l+m}{2}\times\frac{l^{\lambda}+m^{\lambda}}{2} = \frac{l^{\lambda+1}+m^{\lambda+1}}{4}+\frac{l^{\lambda-1}+m^{\lambda-1}}{4}$; sed, quemadmodùm per hoc lemma est $\frac{l^{\lambda}+m^{\lambda}}{2}$ = cos. λA, erit $\frac{l^{\lambda+1}+m^{\lambda+1}}{2}$ = cos. $\overline{\lambda+1}\times$A = cos. A + B, atque $\frac{l^{\lambda-1}+m^{\lambda-1}}{2}$ = cos. $\overline{\lambda-1}\times$A = cos. $\overline{B-A}$, adeoque cos. A $\times$ cos. B $= \frac{1}{2}$ cos. $\overline{A+B}+\frac{1}{2}$ cos. $\overline{B-A}$.

Atque hoc calculi methodo facilè eruuntur sequentes formulæ pro duobus angulis A et B, advertendo esse cos. $\overline{B-A}$ = cos. $\overline{A-B}$, sin. $\overline{B-A}$ = − sin. $\overline{A-B}$, et cos. o = 1.

1°. Cos. A $\times$ cos. B $= \frac{1}{2}$ cos. $\overline{A+B}+\frac{1}{2}$ cos. $\overline{A-B}$.
2°. Sin. A $\times$ sin. B $= -\frac{1}{2}$ cos. $\overline{A+B}+\frac{1}{2}$ cos. $\overline{A-B}$.
3°. Sin. A $\times$ cos. B $= \frac{1}{2}$ sin. $\overline{A+B}+\frac{1}{2}$ sin. $\overline{A-B}$.

Atque ex illis hæ sequentes eliciuntur,

4°. Cos. $\overline{A+B}$ = cos. A $\times$ cos. B − sin. A $\times$ sin. B.
5°. Cos. A − B = sin. A $\times$ sin. B + cos. A $\times$ cos. B.
6°. Sin. A + B = sin. A $\times$ cos. B + cos. A $\times$ sin. B.
7°. Sin. A − B = sin. A $\times$ cos. B − cos. A $\times$ sin. B.

Tùm ex his valores tangentium haud ægrè derivantur,

Quippe

Quippe cum ſit generatim pro quovis angulo A, $\text{tang. } A = \frac{\text{ſin. } A}{\text{coſ. } A}$, erit $\text{tang. } \overline{A+B} = \frac{\text{ſin. } \overline{A+B}}{\text{coſ. } \overline{A+B}} = \frac{\text{ſin. } A \times \text{coſ. } B + \text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{coſ. } B - \text{ſin. } A \times \text{ſin. } B} = \frac{\text{ſin. } A \times \text{coſ. } B + \text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{ſin. } B.}$ $\times \frac{\text{coſ. } A \times \text{ſin. } B}{\text{coſ. } A \times \text{coſ. } B - \text{ſin. } A \times \text{ſin. } B} = \frac{\text{tang. } A}{\text{tang. } B} + 1$ $\times \frac{1}{\frac{1}{\text{tang. } B} - \text{tang. } A} = \frac{\text{tang. } A + \text{tang. } B}{1 - \text{tang. } A \times \text{tang. } B}$. Simili calculo prodit $\text{tang. } \overline{A-B} = \frac{\text{tang. } A - \text{tang. } B}{1 + \text{tang. } A \times \text{tang. } B}$. Unde ſtatui poſſunt,

1°. $\text{Tang. } \overline{A+B} = \frac{\text{tang. } A + \text{tang. } B}{1 - \text{tang. } A \times \text{tang. } B}$.

2°. $\text{Tang. } \overline{A-B} = \frac{\text{tang. } A - \text{tang. } B}{1 + \text{tang. } A \times \text{tang. } B}$.

3°. $\text{Tang. } A \times \text{tang. } B = \frac{\text{tang. } \overline{A+B} - \text{tang. } A - \text{tang. } B}{\text{tang. } \overline{A+B}}$,

vel $\text{tang. } A \times \text{tang. } B = \frac{\text{tang. } A - \text{tang. } B - \text{tang. } \overline{A-B}}{\text{tang. } \overline{A-B}}$.

COROLL. II.

Erat in lemmate $\dot{A} = \frac{\dot{x}}{x\sqrt{-1}}$, unde eſt $A\sqrt{-1} = \log. x$.

Denotet igitur E numerum cujus logarithmus hyperbolicus eſt 1, eritque $E^{A\sqrt{-1}} = x$, et cum ſit $x = c \pm \sqrt{cc - 1}$, inde obtinetur $c = \text{coſ. } A = \frac{E^{A\sqrt{-1}} + E^{-A\sqrt{-1}}}{2}$, atque $\text{ſin. } A = \frac{E^{A\sqrt{-1}} - E^{-A\sqrt{-1}}}{2\sqrt{-1}}$.

 Sunt

Sunt qui his ſinuum et coſinuum valoribus potiùs utuntur; verùm ii valores, quos exhibet corollarium præcedens, ſimpliciores ſunt et calculo plerumque aptiores.

Coroll. III.

Quoniam eſt $2 \times \text{coſ.}\, A = l + m$, erit

$$2^{\lambda} \times \overline{\text{coſ.}\, A}|^{\lambda} = \begin{cases} l^{\lambda} + \lambda l^{\lambda-1} m + \lambda \times \frac{\lambda - 1}{2} l^{\lambda-2} m^{2} + \lambda \\ \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} l^{\lambda-3} m^{3} +, \text{\&c.} \\ m^{\lambda} + \lambda m^{\lambda-1} l + \lambda \times \frac{\lambda-1}{2} m^{\lambda-2} l^{2} + \lambda \\ \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} m^{\lambda-3} l^{3} +, \text{\&c.} \end{cases}$$

aſſumendo ſcilicet primos et ultimos terminos homologos ſeriei exprimentis quantitatem $\overline{l+m}|^{\lambda}$: unde, propter $lm = 1$, provenit

$$2^{\lambda-1} \times \overline{\text{coſ.}\, A}|^{\lambda} = \frac{l^{\lambda}+m^{\lambda}}{2} + \lambda \times \frac{l^{\lambda-2}+m^{\lambda-2}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{l^{\lambda-4}+m^{\lambda-4}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \times \frac{l^{\lambda-6}+m^{\lambda-6}}{2} +, \text{\&c.}$$

atque adeò per lemma

$$\overline{\text{coſ.}\, A}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in coſ.}\, \lambda A + \lambda \text{ coſ.}\, \overline{\lambda - 2} \times A + \lambda \times \frac{\lambda-1}{2} \text{coſ.}\, \overline{\lambda-4} \times A + \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \text{ coſ.}\, \overline{\lambda-6} \times A +, \text{\&c.}$$

Ubi λ eſt numerus impar, terminus ultimus ſeriei erit ille in quo numerus λ, vel $\lambda - 2$, vel $\lambda - 4$, &c. qui multiplicat angulum A, evadit æqualis 1. Ubi verò λ eſt numerus par, terminus ultimus ſeriei erit ille in quo numerus prædictus evadit æqualis o, quo in casu semissis tantùm ultimi termini sumenda est; cùm enim series hæc colligatur ex numero pari terminorum homologorum, quæ tamen, ubi λ est numerus par, constare debet ex terminorum numero impari, ideò duplum exhibet terminum ultimum.

Simili modo cùm sit $2 \times \text{sin. } A = \overline{l - m} \times \sqrt{-1}$, erit

$$2^{\lambda} \times \overline{\text{sin. } A}|^{\lambda} = \sqrt{-1} \times \begin{cases} l^{\lambda} - \lambda l^{\lambda-1} m + \lambda \times \frac{\lambda-1}{2} l^{\lambda-2} m^2 \\ \quad - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} l^{\lambda-3} m^3 +, \\ \quad \&c. \\ \pm m^{\lambda} \mp \lambda m^{\lambda-1} l \pm \lambda \times \frac{\lambda-1}{2} m^{\lambda-2} l^2 \\ \quad \mp \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} m^{\lambda-3} l \pm, \\ \quad \&c. \end{cases}$$

Terminis inferioribus hujus seriei præfiguntur alternatim signa $+ -$ ubi λ est numerus par, et signa $- +$ ubi λ est numerus impar, adeoque in priore casu est

$$2^{\lambda-1} \times \overline{\text{sin. } A}|^{\lambda} = \overline{\sqrt{-1}}|^{\lambda} \text{ in } \frac{l^{\lambda} + m^{\lambda}}{2} - \lambda \times \frac{l^{\lambda-2} + m^{\lambda-2}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{l^{\lambda-4} + m^{\lambda-4}}{2} - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \times \frac{l^{\lambda-6} + m^{\lambda-6}}{2} +, \&c.$$

et in casu posteriori

$$2^{\lambda-1} \times \overline{\text{sin. } A}|^{\lambda} = \overline{\sqrt{-1}}|^{\lambda} \text{ in } \frac{l^{\lambda} - m^{\lambda}}{2} - \lambda \times \frac{l^{\lambda-2} - m^{\lambda-2}}{2} + \lambda \times \frac{\lambda-1}{2} \times \frac{l^{\lambda-4} - m^{\lambda-4}}{2} - \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \times \frac{l^{\lambda-6} - m^{\lambda-6}}{2} +, \&c.$$

Adeoque si λ sit numerus par, erit

$$\overline{\text{sin. } A}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in } \pm \text{ cos. } \lambda A \mp \lambda \text{ cos. } \overline{\lambda - 2} \times A \pm \lambda \times \frac{\lambda-1}{2} \text{ cos. } \overline{\lambda - 4} \times A \mp \lambda \times \frac{\lambda-1}{2} \times \frac{\lambda-2}{3} \text{ cos. } \overline{\lambda - 6} \times A \pm, \&c.$$

 Signa

Signa hîc alternatim mutantur, et ſuperiora ſunt adhibenda, ubi λ exprimit unum ex numeris 4, 8, 12, 16, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = 1$; inferiora autem adhibenda, ubi λ exprimit unum ex numeris 2, 6, 10, 14, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = -1$.

Si λ ſit numerus impar, cùm per lemma ſit $\frac{l^{\lambda} - m^{\lambda}}{2}\sqrt{-1} = \text{ſin. } \lambda A$, et $\frac{l^{\lambda-2} - m^{\lambda-2}}{2}\sqrt{-1} = \text{ſin. } \overline{\lambda - 2} \times A$, &c. habetur

$$\overline{\text{ſin. } A}|^{\lambda} = \frac{1}{2^{\lambda-1}} \text{ in } \pm \text{ſin. } \lambda A \mp \lambda \times \text{ſin. } \overline{\lambda - 2} \times A \pm \lambda \times \frac{\lambda - 1}{2} \text{ſin. } \overline{\lambda - 4} \times A \mp \lambda \times \frac{\lambda - 1}{2} \times \frac{\lambda - 2}{3} \text{ſin. } \overline{\lambda - 6} \times A \pm, \text{ \&c.}$$

ubi ſigna ſuperiora ſunt uſurpanda, cùm λ exprimit unum ex numeris 1, 5, 9, 13, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = \sqrt{-1}$; et ſigna inferiora, cùm λ fuerit unus ex numeris 3, 7, 11, 15, &c. quia tunc eſt $\overline{\sqrt{-1}}|^{\lambda} = \sqrt{-1}$.

Notandum autem, ſeriei ultimum terminum eſſe illum in quo numerus λ, vel λ — 2, vel λ — 4, &c. eſt æqualis 1 ubi λ eſt numerus impar; atque terminum ultimum eſſe illum in quo prædictus numerus eſt æqualis o ubi λ eſt numerus par, quo in caſu ſemiſſis tantùm ultimi termini aſſumenda eſt ob rationem ſuperiùs datam.

Ex his ſinuum et coſinuum expreſſionibus alia hujuſmodi theoremata deducere liceret, ſed quæ hîc traduntur ad præſens inſtitutum ſufficiunt.

COROLL.

COROLL. IV.

Notum eſt fluentem fluxionis $\dot{A}$ coſ. A eſſe ſin. A, atque fluentem fluxionis $\dot{A}$ ſin. A eſſe ſin. verſ. A. Pariter ſi ſumatur arcus λA qui ſit ad arcum A ut numerus quilibet λ ad 1, cùm ſit $\lambda \dot{A}$ coſ. λA æqualis fluxioni ſinûs arcûs λA, erit flu. $\dot{A}$ coſ. $\lambda A =$ $\frac{\text{ſin. } \lambda A}{\lambda}$, et flu. $\dot{A}$ ſin. $\lambda A = \frac{\text{ſin. verſ. } \lambda A}{\lambda}$. Itemque, ſi ad arcum λA adjungatur arcus datus d, cùm fluxio arcûs $\lambda A + d$ ſit æqualis $\lambda \dot{A}$, erit flu. $\dot{A}$ coſ. $\lambda A + d$ $= \frac{\text{ſin. } \overline{\lambda A + d}}{\lambda}$, et flu. $\dot{A}$ ſin. $\overline{\lambda A + d} = \frac{\text{ſin. verſ. } \overline{\lambda A + d}}{\lambda}$.

Sumantur jam duo anguli, vel duo arcus λA et μA, qui ſint ad angulum, vel arcum A reſpectivè, ut λ et μ ad 1, atque per Coroll. II. habetur coſ. λA coſ. $\times \mu A$ $= \frac{1}{2}$ coſ. $\overline{\lambda + \mu} \times A + \frac{1}{2}$ coſ. $\overline{\lambda - \mu} \times A$; unde erit fluens fluxionis $\dot{A}$ coſ. $\lambda A \times$ coſ. μA æqualis $\frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}} + \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

Atque hoc methodo prodeunt ſequentes formulæ

1°. Flu. $\dot{A}$ coſ. $\lambda A \times$ coſ. $\mu A = \frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}}$ $+ \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

2°. Flu. $\dot{A}$ ſin. $\lambda A \times$ ſin. $\mu A = - \frac{\text{ſin. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}}$ $+ \frac{\text{ſin. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

3°. Flu.

3°. Flu. $\dot{A}$ ſin. $\lambda A \times$ coſ. $\mu A = \frac{\text{ſin. verſ. } \overline{\lambda + \mu} \times A}{2 \times \overline{\lambda + \mu}}$

$+ \frac{\text{ſin. verſ. } \overline{\lambda - \mu} \times A}{2 \times \overline{\lambda - \mu}}$.

Advertendum autem eſt, ubi $\lambda = \mu$, tunc eſſe coſ. $\lambda A \times$ coſ. $\mu A = \frac{1}{2}$ coſ. $2 \lambda A + \frac{1}{2}$, ſin. $\lambda A \times$ ſin. $\mu A = - \frac{1}{2}$ coſ. $2 \lambda A + \frac{1}{2}$, ſin. $\lambda A \times$ coſ. $\mu A = \frac{1}{2}$ ſin. $2 \lambda A$; adeoque in hoc caſu formulæ præcedentes evadunt

1°. Flu. $\dot{A} \times \overline{\text{coſ. } \lambda A}|^2 = \frac{\text{ſin. } 2\lambda A}{4\lambda} + \frac{A}{2}$.

2°. Flu. $\dot{A} \times \overline{\text{ſin. } \lambda A}|^2 = - \frac{\text{ſin. } 2\lambda A}{4\lambda} + \frac{A}{2}$.

3°. Flu. $\dot{A} \times$ ſin. $\lambda A \times$ coſ. $\lambda A = \frac{\text{ſin. verſ. } 2\lambda A}{4\lambda}$.

Si angulo λA addatur angulus datus d, erit coſ. $\overline{\lambda A + d} \times$ coſ. $\mu A = \frac{1}{2}$ coſ. $\overline{\overline{\lambda + \mu} \times A + d} + \frac{1}{2}$ coſ. $\overline{\overline{\lambda - \mu} \times A + d}$, atque inde

1°. Flu. $\dot{A}$ coſ. $\overline{\lambda A + d} \times$ coſ. $\mu A = \frac{\text{ſin. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}}$

$+ \frac{\text{ſin. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

2°. Flu. $\dot{A}$ ſin. $\overline{\lambda A + d} \times$ ſin. $\mu A = - \frac{\text{ſin. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}}$

$+ \frac{\text{ſin. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

3°. Flu. $\dot{A}$ ſin. $\overline{\lambda A + d} \times$ coſ. $\mu A = \frac{\text{ſin. verſ. } \overline{\overline{\lambda + \mu} \times A + d}}{2 \times \overline{\lambda + \mu}}$

$+ \frac{\text{ſin. verſ. } \overline{\overline{\lambda - \mu} \times A + d}}{2 \times \overline{\lambda - \mu}}$.

4°. Flu.

4°. Flu. $\dot{A}$ cos. $\overline{\lambda A + d} \times$ sin. $\mu A = \frac{\text{sin. vers. } \overline{\lambda + \mu} \times A + d}{2 \times \overline{\lambda + \mu}}$

$- \frac{\text{sin. vers. } \overline{\lambda - \mu} \times A + d}{2 \times \overline{\lambda - \mu}}$.

Si fuerit $\lambda = \mu$, erit cos. $\overline{\lambda A + d} \times$ cos. $\lambda A =$ $\frac{1}{2}$ cos. $\overline{2\lambda A + d} + \frac{1}{2}$ cos. d, &c. adeoque formulæ præcedentes in has abeunt,

1°. Flu. $\dot{A}$ cos. $\overline{\lambda A + d} \times$ cos. $\lambda A = \frac{\text{sin. } \overline{2\lambda A + d}}{4\lambda}$ $+ \frac{\text{cos. } d}{2} A$.

2°. Flu. $\dot{A}$ sin. $\overline{\lambda A + d} \times$ sin. $\lambda A = - \frac{\text{sin. } \overline{2\lambda A + d}}{4\lambda}$ $+ \frac{\text{cos. } d}{2} A$.

3°. Flu. $\dot{A}$ sin. $\overline{\lambda A + d} \times$ cos. $\lambda A = \frac{\text{sin. vers. } \overline{2\lambda A + d}}{4\lambda}$ $+ \frac{\text{sin. } d}{2} A$.

4°. Flu. $\dot{A}$ cos. $\overline{\lambda A + d} \times$ sin. $\lambda A = \frac{\text{sin. vers. } \overline{2\lambda A + d}}{4\lambda}$ $- \frac{\text{sin. } d}{2} A$.

PROPOSITIO I. PROBLEMA.

In systemate duorum planetarum circa Solem in orbibus penè circularibus revolventium, requiratur vis planetæ exterioris ad perturbandum motum interioris.

Revolvantur planetæ duo P et Q (Fig. 1.) in eodem plano circa Solem in S, et jungantur SP, SQ, PQ.

Orbis

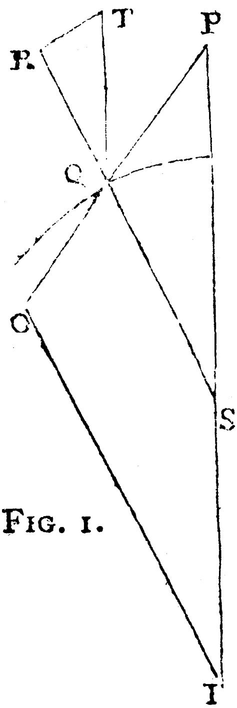

FIG. 1.

Orbis planetæ interioris Q, cujus motus hîc investigamus, circularis supponitur nisi quatenùs mutatur ejus figura vi planetæ P; orbem verò planetæ P ut accuratè circularem habemus. Positâ ergò unitate pro distantiâ corporis Q à Sole ubi ambo planetæ versantur in conjunctione cum Sole, fiant SQ $= x$, PQ $= z$, SP $= k$; tumque vis attractionis Solis in distantiâ æquali 1 sit ad vim attractionis planetæ P in eâdem distantiâ ut 1 ad ϕ, eritque $\frac{\phi}{z^2}$ gravitas planetæ Q in planetam P. Producatur jam, si opus est, PQ ad O ut sit PO $= \frac{\phi}{z^2}$, et ductâ OI parallelâ ipsi QS occurrente rectæ PS productæ in I, propter triangula similia PQS, POI, erit PQ · PS :: PO · PI, hoc est, PI $= \frac{\phi k}{z^3}$, atque PQ · QS :: PO · OI, hoc est, OI $= \frac{\phi x}{z^3}$. Sed, quia parùm differt x ab unitate et admodùm exigua est vis ϕ, pro x scribi potest 1 in omnibus iis terminis qui ducuntur in ϕ, adeoque OI $= \frac{\phi}{z^3}$. Ex vi PI auferatur vis $\frac{\phi}{k^2}$ qua gravitat Sol in planetam P, et vis residua $\frac{\phi k}{z^3} - \frac{\phi}{k^2}$ est ea qua perturbatur motus planetæ Q in directione parallelâ rectæ PS: nàm cùm motus planetarum

planetarum referantur ad Solem ſpectatum tanquàm immotum, vis $\frac{\varphi k}{z^3}$ pars ea $\frac{\varphi}{k^2}$, qua ſimul urgentur Sol et planeta Q verſus P ſecundum lineas parallelas, non mutat corporum S et Q ſitum ad ſe invicem, ideoque differentia virium ſola perturbationem inducit.

Quare differentia illa, nimirùm $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$, exponatur per lineam QT parallelam rectæ PS, et in SQ demiſſo perpendiculo TR, vis QT reſolvetur in vires TR, QR, eritque vis QT ad vim TR ut radius 1 ad ſinum anguli QSP, adeoque vis TR $= \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. QSP, et vis QR $= \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. QSP. Ex vi autem QR tollatur vis OI utpotè in contrarium agens, et manebit vis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. QSP $- \frac{\varphi}{z^3}$. Vires igitur, quibus planeta P perturbat motum planetæ Q quatenùs in eodem plano moventur, ſunt

1°. Vis TR ad radium QS perpendicularis, qua augetur vel minuitur area tempore dato deſcripta, eſtque æqualis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. QSP.

2°. Vis $\frac{\varphi}{z^3} \times \overline{k \text{ coſ. QSP} - 1} - \frac{\varphi}{k^2}$ coſ. QSP, qua retrahitur planeta Q à Sole in directione radii SQ.

Ut autem harumce virium expreſſiones formam induant calculo accommodam, ope trianguli PSQ habebitur $\overline{PQ}^2 = zz = kk + xx - 2kx \times$ coſ. QSP, ſive, poſitâ $x = 1$ ob rationem dictam, $zz = kk$

 + 1

$+ 1 - 2k \times$ cos. QSP. Assumatur jam angulus s qui semper sit ad angulum QSP in ratione n ad 1, eritque QSP $= \frac{1}{n}s$, et posito $kk + 1 = tt$, et $\frac{2k}{t^2} = b$, erit $z^2 = t^2 \times \overline{1 - b \text{ cos. } \frac{1}{n}s}$, hincque $\frac{1}{z^3} = \frac{1}{t^3} \times \overline{1 - b \text{ cos. } \frac{1}{n}s}|^{-\frac{3}{2}}$. Si b fuerit unitati ferè æqualis, et evolvatur quantitas $\overline{1 - b \text{ cos. } \frac{1}{n}s}|^{-\frac{3}{2}}$ in seriem modo solito, series illa parùm convergit, estque ad operationes analyticas minùs commoda. Series igitur alia investiganda est, et quia ex lemmate patet hujusmodi quantitatem $\overline{\text{cos. A}}|^{\lambda}$ exprimi posse aggregato terminorum, quorum singuli ducuntur in cosinus angulorum qui sunt anguli A multiplices, generatim supponemus $\overline{1 - b \text{ cos. } \frac{1}{n}s}|^{m} = \text{R} + \text{S cos. } \frac{1}{n}s + \text{T cos. } \frac{2}{n}s + \text{V cos. } \frac{3}{n}s + \text{W cos. } \frac{4}{n}s +$, &c.

Atque ut inveniantur valores coefficientium R, S, T, &c. sumatur utrinque fluxio, nempe $\frac{mb}{n}\dot{s} \times \text{sin. } \frac{1}{n}s \times \overline{1 - b \text{ cos. } \frac{1}{n}s}|^{m-1} = -\text{S} \times \frac{1}{n}\dot{s} \times \text{sin. } \frac{1}{n}s - \text{T} \times \frac{2}{n}\dot{s} \times \text{sin. } \frac{2}{n}s - \text{V} \times \frac{3}{n}\dot{s} \times \text{sin. } \frac{3}{n}s - \text{W} \times \frac{4}{n}\dot{s} \times \text{sin. } \frac{4}{n}s -$, &c. atque ducatur æquatio hæc in $\overline{1 - b \text{ cos. } \frac{1}{n}s}$, et substituto pro $\overline{1 - b \text{ cos. } \frac{1}{n}s}|^{m}$ ipsius valore $\text{R} + \text{S cos. } \frac{1}{n}s + \text{T cos. } \frac{2}{n}s +$, &c. fiet $mb \times \text{sin. } \frac{1}{n}s$

$\times$

$$\overline{\overline{\times R + S \text{ cos.} \tfrac{1}{n}s + T \text{ cos.} \tfrac{2}{n}s + V \text{ cos.} \tfrac{3}{n}s +, \&c. = 1 - b \text{ cos.} \tfrac{1}{n}s}}$$

$$\times - S \times \text{sin.} \tfrac{1}{n}s + 2T \times \text{sin.} \tfrac{2}{n}s \times 3V \times \text{sin.} \tfrac{3}{n}s + W \times \text{sin.} \tfrac{4}{n}s +, \&c.$$

et factâ multiplicatione, cùm ſit (per Coroll. I. Lem.)

$\text{ſin.} \tfrac{1}{n}s \times \text{coſ.} \tfrac{r}{n}s = \tfrac{1}{2} \text{ſin.} \tfrac{r+1}{n}s - \tfrac{1}{2} \text{ſin.} \tfrac{r-1}{n}s$, ac

$\text{ſin.} \tfrac{r}{n}s \times \text{coſ.} \tfrac{1}{n}s = \tfrac{1}{2} \text{ſin.} \tfrac{r+1}{n}s + \tfrac{1}{2} \text{ſin.} \tfrac{r-1}{n}s$, emerget

$$\left.\begin{matrix} +2mbR \\ +2S \\ -2bT \\ -mbT \end{matrix}\right\} \times \text{ſin.} \tfrac{1}{n}s \left\{\begin{matrix} +mbS \\ -bS \\ +4T \\ -3bV \\ -mbV \end{matrix}\right\} \times \text{ſin.} \tfrac{2}{n}s \left\{\begin{matrix} +mbT \\ -2bT \\ +6V \\ -4bW \\ -mbW \end{matrix}\right\} \times \text{ſin.} \tfrac{3}{n}s, \&c. = 0$$

Deinde nihilo æquando ſingulos terminos, prodeunt $T = \dfrac{2S + 2mbR}{\overline{m+2} \times b}$, $V = \dfrac{4T + \overline{m-1} \times bS}{\overline{m+3} \times b}$, $W = \dfrac{6V + \overline{m-2} \times bT}{\overline{m+4} \times b}$, &c. quorum valorum progreſſus ſatis manifeſtus eſt.

Datis igitur primis duobus coefficientibus R et S, dabuntur et reliqui: R et S autem ſic inveniuntur.

Eſt $\overline{1 - b \text{ coſ.} \tfrac{1}{n}s}\Big|^{m} = 1 - mb \text{ coſ.} \tfrac{1}{n}s + m \times \tfrac{m-1}{2} b^2 \overline{\text{coſ.} \tfrac{1}{n}s}\Big|^{2} - m \times \tfrac{m-1}{2} \times \tfrac{m-2}{3} b^3 \overline{\text{coſ.} \tfrac{1}{n}s}\Big|^{3}$, &c. $= R + S \text{ coſ.} \tfrac{1}{n}s + T \text{ coſ.} \tfrac{2}{n}s + V \text{ coſ.} \tfrac{3}{n}s +$, &c. Evolvantur termini $\overline{\text{coſ.} \tfrac{1}{n}s}\Big|^{2}$, $\overline{\text{coſ.} \tfrac{1}{n}s}\Big|^{4}$, $\overline{\text{coſ.} \tfrac{1}{n}s}\Big|^{6}$, &c. per methodum traditam in Coroll. III. Lem. ac, collectis ſimul omnibus terminis qui nullo coſinu afficiuntur, prodibit

 R =

$$R = 1 + \frac{m}{2} \times \frac{m-1}{2} b^2 + \frac{m}{2} \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} b^4$$
$$+ \frac{m}{2} \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} \times \frac{m-4}{6} \times \frac{m-5}{6} b^6 +, \text{ \&c.}$$

cujus ſeriei progreſſio ſatis patet; atque adeò, cum ſit in hoc noſtro problemate $m = -\frac{3}{2}$, erit

$$R = 1 + \frac{3 \times 5}{4 \times 4} b^2 + \frac{3 \times 5}{4 \times 4} \times \frac{7 \times 9}{8 \times 8} b^4 + \frac{3 \times 5}{4 \times 4} \times \frac{7 \times 9}{8 \times 8}$$
$$\times \frac{11 \times 13}{12 \times 12} b^6 + \frac{3 \times 5}{4 \times 4} \times \frac{7 \times 9}{8 \times 8} \times \frac{11 \times 13}{12 \times 12} \times \frac{15 \times 17}{16 \times 16} b^8 +, \text{ \&c.}$$

Inſpicienti indolem hujus ſeriei patebit terminum quemlibet æquari termino antecedenti ducto in $\frac{\overline{r+1} \times \overline{r-1}}{r^2} b^2$, ſive $\frac{r^2-1}{r^2} b^2$, r exiſtente æquali numero quadruplicato terminorum præcedentium: ſic terminus ſextus, quia habetur in hoc caſu $r = 5 \times 4 = 20$, æqualis eſt termino quinto $\frac{3 \times 5}{4 \times 4} \ldots\ldots \frac{15 \times 17}{16 \times 16} b^8$ ducto in $\frac{19 \times 21}{20 \times 20} b^2$.

Termino igitur quovis hujus ſeriei dicto B, terminus ſubſequens erit $Bb^2 \times \frac{r^2-1}{r^2}$: et manente deinceps eodem, quem in hoc termino habet, numeri r valore, termini ſubſequentes erunt, $Bb^4 \times \frac{r^2-1}{r^2} \times \frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}$, $Bb^6 \times \frac{r^2-1}{r^2} \times \frac{\overline{r+4}|^2-1}{\overline{r+4}|^2} \times \frac{\overline{r+8}|^2-1}{\overline{r+8}|^2}$, $Bb^8 \times \frac{r^2-1}{r^2} \ldots\ldots \frac{\overline{r+12}|^2-1}{\overline{r+12}|^2}$, &c. Sed eſt $\frac{r^2-1}{r^2} = 1 - \frac{1}{r^2}$, $\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2} = 1 - \frac{1}{\overline{r+4}|^2}$, &c. et ſi fuerit r

numerus

numerus aliquantùm magnus, erit $\frac{r^2-1}{r^2}\times\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}$ $=1-\frac{1}{r^2}-\frac{1}{\overline{r+4}|^2}$, et $\frac{r^2-1}{r^2}\times\frac{\overline{r+4}|^2-1}{\overline{r+4}|^2}\times\frac{\overline{r+8}|^2-1}{\overline{r+8}|^2}$ $=1-\frac{1}{r^2}-\frac{1}{\overline{r+4}|^2}-\frac{1}{\overline{r+8}|^2}$, atque ita porrò, rejiciendo fractiones hujus generis $\frac{1}{r^2\times\overline{r+4}|^2}$ et alias his minores.

Unde termini omnes prædicti, incipiendo à termino B, erunt

$$
\begin{array}{rcl}
B+Bb^2+Bb^4+Bb^6+Bb^8+,\ \&c. & = & B\times\frac{1}{1-b^2} \\
-\frac{Bb^2}{r^2}-\frac{Bb^4}{r^2}-\frac{Bb^6}{r^2}-\frac{Bb^8}{r^2}-,\ \&c. & = & -\frac{B}{r^2}\times\frac{b^2}{1-b^2} \\
-\frac{Bb^4}{\overline{r+4}|^2}-\frac{Bb^6}{\overline{r+4}|^2}-\frac{Bb^8}{\overline{r+4}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+4}|^2}\times\frac{b^4}{1-b^2} \\
-\frac{Bb^6}{\overline{r+8}|^2}-\frac{Bb^8}{\overline{r+8}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+8}|^2}\times\frac{b^6}{1-b^2} \\
-\frac{Bb^8}{\overline{r+12}|^2}-,\ \&c. & = & -\frac{B}{\overline{r+12}|^2}\times\frac{b^8}{1-b^2} \\
\&c. & & \&c.
\end{array}
$$

ac proinde tandem fit

$$
R=1+\frac{3\times5}{4\times4}b^2+\frac{3\times5}{4\times4}\times\frac{7\times9}{8\times8}b^4+\frac{3\times5}{4\times4}\times\frac{7\times9}{8\times8}\times\frac{11\times13}{12\times12}b^6\times\frac{3\times5}{4\times4}\ldots\frac{15\times17}{16\times16}b^8+,\ \&c.+\frac{B}{1-b^2}
$$
$$
\times\,\overline{1-\frac{b^2}{r^2}-\frac{b^4}{\overline{r+4}|^2}-\frac{b^6}{\overline{r+8}|^2}-\frac{b^8}{\overline{r+12}|^2}-\frac{b^{10}}{\overline{r+16}|^2}-,\ \&c.}
$$

Unde si, computatis, exempli gratiâ, decem terminis, undecimus designetur per B, erit $r=10\times4=40$, et summa illorum decem terminorum addita summæ

ſummæ ſeriei $\frac{B}{1-b^2} \times \overline{1 - \frac{b^2}{r^2} - \frac{b^4}{\overline{r \times 4}|^2} -}$, &c. dabit valorem ipſius R.

Simili modo ſi in æquatione prædictâ $1 - mb \text{ coſ.} \frac{1}{n} s + m \times \frac{m-1}{2} b^2 \times \overline{\text{coſ.} \frac{1}{n} s}|^2 + m \times \frac{m-1}{2} \times \frac{m-2}{3} b^3 \times \overline{\text{coſ.} \frac{1}{n} s}|^3 +$, &c. $= R + S \text{ coſ.} \frac{1}{n} s + T \text{ coſ.} \frac{2}{n} s + V \text{ coſ.} \frac{3}{n} s +$, &c. evolvantur quantitates $\overline{\text{coſ.} \frac{1}{n} s}|^3$. $\overline{\text{coſ.} \frac{1}{n} s}|^5$, $\overline{\text{coſ.} \frac{1}{n} s}|^7$, &c. in ſuos valores, prout in Coroll. III. Lem. edoctum eſt, et colligantur omnes termini qui ducuntur in coſ. $\frac{1}{n} s$ exurget

$$S = -mb - m \times \frac{m-1}{2} \times \frac{m-2}{4} b^3 - m \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} \times \frac{m-4}{6} b^5 - m \times \frac{m-1}{2} \times \frac{m-2}{4} \times \frac{m-3}{4} \times \frac{m-4}{6} \times \frac{m-5}{6} \times \frac{m-6}{8} b^7 -, \text{ \&c.}$$

ſive, poſito $m = -\frac{3}{2}$,

$$S = \tfrac{3}{2} b + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} b^3 + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} \times \frac{9 \times 11}{8 \times 12} b^5 + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} \times \frac{9 \times 11}{8 \times 12} \times \frac{13 \times 15}{12 \times 16} b^7 + \tfrac{3}{2} \cdots\cdots \frac{13 \times 15}{12 \times 16} \times \frac{17 \times 19}{16 \times 20} b^9 +, \text{ \&c.}$$

Patet autem terminum quemlibet hujus ſeriei æquari termino antecedenti ducto in $\frac{\overline{r+1} \times \overline{r+3}}{r \times \overline{r+4}} b^2$, exiſtente r æquali numero terminorum præcedentium quadruplicato: ſic terminus ſextus, quia tunc $r = 5 \times 4 = 20$,

 eſt

eſt æqualis termino quinto $\frac{3}{2} \cdots\cdots \frac{17 \times 19}{16 \times 20} b^9$ ducto in $\frac{21 \times 23}{20 \times 24} b^2$. Quamobrem termino quovis hujus ſeriei dicto B, terminus ſubſequens erit $Bb^2 \times \frac{\overline{r+1} \times \overline{r+3}}{r \times \overline{r+4}}$, ſive $Bb^2 \times 1 + \frac{3}{r \times \overline{r+4}}$, et manente jam eodem valore numeri r, termini reliqui erunt, $Bb^4 \times \overline{1 + \frac{3}{r \times \overline{r+4}}} \times \overline{1 + \frac{3}{\overline{r+4} \times \overline{r+8}}}$, $Bb^6 \times \overline{1 + \frac{3}{r \times \overline{r+4}}} \times \overline{1 + \frac{3}{\overline{r+4} \times \overline{r+8}}} \times \overline{1 + \frac{3}{\overline{r+8} \times \overline{r+12}}}$, &c. Sed ſi fuerit r numerus aliquantùm magnus, erit $\overline{1 + \frac{3}{r \times \overline{r+4}}} \times \overline{1 + \frac{3}{\overline{r+4} \times \overline{r+8}}} = 1 + \frac{3}{r \times \overline{r+4}} + \frac{3}{\overline{r+4} \times \overline{r+8}}$ quamproximè, et $\overline{1 + \frac{3}{r \times \overline{r+4}}} \times \overline{1 + \frac{3}{\overline{r+4} \times \overline{r+8}}} \times \overline{1 + \frac{3}{\overline{r+8} \times \overline{r+12}}} = 1 + \frac{3}{r \times \overline{r+4}} + \frac{3}{\overline{r+4} \times \overline{r+8}} + \frac{3}{\overline{r+8} \times \overline{r+12}}$, &c. Unde termini omnes prædicti incipientes à termino B erunt

$$B + Bb^2 + Bb^4 + Bb^6 + Bb^8 + \text{, \&c.} = \frac{B}{1-b^2}$$

$$+ \frac{3Bb^2}{r \times \overline{r+4}} + \frac{3Bb^4}{r \times \overline{r+4}} + \frac{3Bb^6}{r \times \overline{r+4}} + \frac{3Bb^8}{r \times \overline{r+4}} + \text{, \&c.} = \frac{3B}{r \times \overline{r+4}} \times \frac{b^2}{1-b^2}$$

$$+ \frac{3Bb^4}{\overline{r+4} \times \overline{r+8}} + \frac{3Bb^6}{\overline{r+4} \times \overline{r+8}} + \frac{3Bb^8}{\overline{r+4} \times \overline{r+8}} + \text{, \&c.} = \frac{3B}{\overline{r+4} \times \overline{r+8}} \times \frac{b^4}{1-b^2}$$

$$+ \frac{3Bb^6}{\overline{r+8} \times \overline{r+12}} + \frac{3Bb^8}{\overline{r+8} \times \overline{r+12}} + \text{, \&c.} = \frac{3B}{\overline{r+8} \times \overline{r+12}} \times \frac{b^6}{1-b^2}$$

$$+ \frac{3Bb^8}{\overline{r+12} \times \overline{r+16}} + \text{, \&c.} = \frac{3B}{\overline{r+12} \times \overline{r+16}} \times \frac{b^8}{1-b^2}$$

&c. &c.

Ac

Ac proinde erit

$$S = \tfrac{3}{2} b + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} b^3 + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} \times \frac{9 \times 11}{8 \times 12} b^5 + \tfrac{3}{2} \times \frac{5 \times 7}{4 \times 8} \times \frac{9 \times 11}{8 \times 12} \times \frac{13 \times 15}{12 \times 16} b^7 +, \&c. + \frac{B}{1 - b^2}$$

$$\times 1 + \frac{3b^2}{r \times \overline{r+4}} + \frac{3b^4}{\overline{r+4} \times \overline{r+8}} + \frac{3b^6}{\overline{r+8} \times \overline{r+12}} + \frac{3b}{\overline{r+12} \times \overline{r+16}} +, \&c.$$

Itaque ſi, computatis, exempli gratiâ, quindecim terminis, decimus ſextus deſignetur per B, erit $r = 15 \times 4 = 60$, et ſumma terminorum quindecim illorum addita ſummæ ſeriei $\frac{B}{1-b^2} \times 1 + \frac{3b^2}{r \times \overline{r+4}} + \frac{3b^4}{\overline{r+4} \times \overline{r+8}} +$, &c. dabit valorem coefficientis S.

Determinatis hoc pacto quantitatibus aſſumptis R, S, T, &c. jam ut ad expreſſiones virium revertamur, vis TR ad radium QS perpendicularis erat $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. QSP; ſed poſuimus angulum $QSP = \frac{1}{n} s$, eſtque $\frac{1}{z^3} = \frac{1}{t^3}$ in $R + S$ coſ. $\frac{1}{n} s + T$ coſ. $\frac{2}{n} s + V$ coſ. $\frac{3}{n} s + W$ coſ. $\frac{4}{n} s +$, &c.

Unde vis $TR = \frac{\varphi k}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times$ ſin. $\frac{1}{n} s + \frac{S-V}{2}$ ſin. $\frac{2}{n} s + \frac{T-W}{2}$ ſin. $\frac{3}{n} s + \frac{V-X}{2}$ ſin. $\frac{4}{n} s +$, &c.

Et vis quæ planetam Q diſtrahit à Sole in directione radii QS erat $\frac{\varphi}{z^3} \times \overline{k \text{ coſ. } QSP - 1} - \frac{\varphi}{t^3}$ coſ. QSP, hoc eſt, $\frac{\varphi}{t^3}$ in $\frac{kS}{2} - R + \overline{kR + \frac{kT}{2} - \frac{t^3}{k^2} - S}$

× coſ.

$\times$ cof. $\frac{1}{n}s + \frac{kS+kV-2T}{2} \times$ cof. $\frac{2}{n}s + \frac{kT+kW-2V}{2}$ cof. $\frac{3}{n}s + \frac{kV+kX-2W}{3}$ cof. $\frac{4}{n}s +$, &c. Q. E. I.

Propositio II. Problema.

Inæqualitates motûs planetæ interioris ex viribus prædictis ortas inveſtigare.

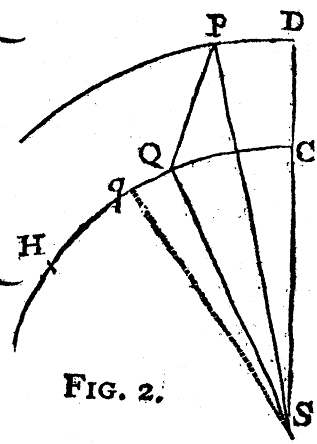

Fig. 2.

Exeant ſimul planetæ P, Q (Fig. 2.) de locis D, C, ubi jacebant in eâdem rectâ cum Sole poſito in S, et poſt aliquod temporis ſpatium reperiantur in P et Q, et jungantur SP, SQ, PQ. Eſto CS = 1, et arcus circularis CQ ſive angulus CSQ = s; denotent prætereà P et Q reſpectivè tempora periodica planetarum P et Q, eritque ang. QSC : ang. PSD :: P : Q, adeoque angulus QSP : ang. QSC :: P — Q : P, unde ang. QSP $= \frac{1}{n}s$, poſito $n = \frac{P}{P-Q}$. Vis attractionis Solis ad diſtantiam QS, et tempus quo corpus, eâdem vi uniformiter agente, impulſum acquirere poſſet eam velocitatem, qua planeta Q in circulo CQ revolvitur, tùm illa ipſa velocitas, exponantur ſigillatim per unitatem; et ſi, ſumpto arcu CH = CS = 1, CH exprimat tempus illud unitati æquale, arcus quilibet quàm minimus Qq exprimet tempus quo uniformi illâ velocitate deſcribitur.

 Unde,

Unde, cum velocitates viribus quibusvis constantibus genitæ sint ut ipsæ vires et tempora, quibus hæ velocitates generantur, conjunctim; erit velocitas 1 planetæ Q in circulo CQ revolventis ad incrementum vel decrementum velocitatis vi Z genitum (scripto nempe Z pro vi planetæ P normaliter ad radium QS agente, prout est in propositione præcedente definita) quo tempore planeta Q describit arcum quàm minimum Qq, ut vis attractionis Solis 1 ducta in tempus CH sive 1, ad vim Z ductam in tempus descriptionis arcûs Qq sive in ipsum in arcum Qq: adeoque incrementum vel decrementum velocitatis vi Z genitum, quo tempore describitur arcus Qq, exprimetur per $Z \times Qq$ sive $Z \times \dot{s}$.

Est autem $Z = \frac{\varphi k}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{sin.} \frac{1}{n} s + \frac{S - V}{2} \text{sin.} \frac{2}{n} s + \frac{T - W}{2} \text{sin.} \frac{3}{n} s +$, &c. et hac quantitate ductâ in $\dot{s}$, tùm sumptâ fluente, prodit velocitatis accceleratio sive retardatio, quam voco U, genita quo tempore describitur à planeta Q arcus CQ, æqualis $\frac{\varphi k n}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{sin. vers.} \frac{1}{n} s + \frac{S - V}{4}$ sin. vers. $\frac{2}{n} s + \frac{T - W}{6}$ sin. vers. $\frac{3}{n} s + \frac{V - X}{8}$ sin. vers. $\frac{4}{n} s +$, &c. sive posito $b = R - \frac{t^3}{k^3} - \frac{T}{2} + \frac{S - V}{4} + \frac{T - W}{6} + \frac{V - X}{8} +$, &c. $U = \frac{\varphi k n}{t^3}$ in $b - \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{cos.} \frac{1}{n} s - \frac{S - V}{4} \text{cos.} \frac{2}{n} s - \frac{T - W}{6} \text{cos.} \frac{3}{n} s - \frac{V - X}{8} \text{cos.} \frac{4}{n} s -$, &c.

Hoc

Hoc pacto obtinetur variatio velocitatis in hypothesi quòd revolvatur planeta Q semper ad eamdem distantiam à Sole, quod in præcedenti calculo supponi potest, cùm tantillùm varietur distantia S Q actione planetæ P.

Hoc facto, ut investigetur variatio distantiæ planetæ Q à Sole, fingamus planetam descripsisse, non arcum circularem C Q, sed arcum curvæ C r (Fig. 3.) et reperiri in puncto r ubi radius S Q productus secat curvam.

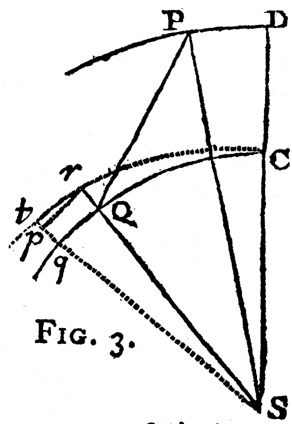

Fig. 3.

Ducatur recta S t viciniffima ipsi S Q occurrens circulo et curvæ q et t; tùm centro S et radio S r describatur arcus rp, sitque S $r = x$. Si planeta Q urgeretur solâ vi tendente ad centrum S, describeret areas temporibus proportionales, atque adeò, cùm ipsius velocitas angularis in loco C supponatur esse 1, in loco r foret æqualis $\frac{1}{x}$; sed in illo quem exhibet schema situ planetarum minuitur hæc velocitas quantitate U suprà definitâ, unde velocitas angularis in loco r erit $\frac{1}{x} - U$; et tempus, quo describeretur arcus Q q velocitate 1, est ad tempus quo describitur arcus rp velocitate $\frac{1}{x} - U$, ut Q q ad $\frac{rp}{\frac{1}{x} - U}$, hoc est, ut $\dot{s}$ ad $\frac{x\dot{s}}{\frac{1}{x} - U}$; unde, cùm $\dot{s}$ exprimat ex jam dictis tempus descriptionis arcûs Q q velocitate 1, exprimet quantitas $\frac{xs}{\frac{1}{x}-U}$ tempus quo deſcribitur arcus rp velocitate $\frac{1}{x}-U$. His poſitis, quoniam planetæ Q receſſus à centro vel ad idem acceſſus pendet ex differentiâ virium, centrifugæ ſcilicet et centripetæ, quibus urgetur in Q; ſi hæc differentia virium dicatur P, et $\dot{v}$ denotet velocitatem aſcenſûs vel deſcenſûs planetæ Q ſecundum radium SQ, per idem planè ratiocinium, quod mox uſurpavimus in inveſtigatione velocitatis U, habebitur $\dot{v} = P \times \frac{x\dot{s}}{\frac{1}{x}-U}$.

Quoniam ex hypotheſi planeta Q, ſepoſitâ actione planetæ P, deſcriberet circulum, vires (centripeta et centrifuga) ſibi invicem et unitati forent æquales: exiſtente autem planetâ Q in r, ipſius attractio in Solem eſt $\frac{1}{x^2}$, ex qua auferenda eſt vis ea qua juxta propoſitionem præcedentem diſtrahitur à Sole, nimirùm $\frac{\phi}{t^3}$ in $A + B$ coſ. $\frac{1}{n}s + C$ coſ. $\frac{2}{n}s + D$ coſ. $\frac{3}{n}s + E$ coſ. $\frac{4}{n}s +$, &c. poſitis $A = \frac{kS}{2} - R$, $B = kR + \frac{kT}{2} - \frac{t^3}{k^2} - S$, $C = \frac{kS+kV-2T}{2}$, $D = \frac{kT+kW-2V}{2}$, $E = \frac{kV+kX-2W}{2}$, &c. atque harum virium differentia componit vim centripetam.

Vis autem centrifuga eſt ſemper in ratione duplicatâ areæ temporis momento deſcriptæ directè et triplicatâ diſtantiæ inversè; unde ſi hæc vis fuerit æqualis

1,

1, ubi incepit planeta movere in C, erit æqualis $x^2 \times \overline{\frac{1}{x} - U}|^2 \times \frac{1}{x^3} = \frac{1}{x} \times \overline{\frac{1}{x} - U}|^2$ ubi movetur in r. Differentia igitur inter vim centrifugam et centripetam, qua urgetur planeta in r supra designata per P, est $\frac{1}{x} \times \overline{\frac{1}{x} - U}|^2 - \frac{1}{x^2} + \frac{\phi}{t^3}$

$$\overline{\times A + B \operatorname{cos.} \frac{1}{n} s + C \operatorname{cos.} \frac{2}{n} s + D \operatorname{cos.} \frac{3}{n} s +, \&c.}$$

hincque habetur $\dot{v} = \dot{s} \times \overline{\frac{1}{x} - U} - \frac{\dot{s}}{x \times \overline{\frac{1}{x} - U}} + \frac{\phi}{t^3}$

$$\times \frac{x \dot{s}}{\frac{1}{x} - U} \times \overline{A + B \operatorname{cos.} \frac{1}{n} s + C \operatorname{cos.} \frac{2}{n} s +, \&c.}$$

Vires, quibus perturbatur motus planetæ Q, cùm exprimantur seriebus quorum termini ducuntur in sinum vel cosinum anguli $\frac{1}{n} s$, vel anguli hujus multiplicis, fingemus differentiam inter distantias SQ et Sr exprimi serie simili, ac propterea ponemus $x = 1 - Q + K \operatorname{cos.} \frac{1}{n} s + L \operatorname{cos.} \frac{2}{n} s + M \operatorname{cos.} \frac{3}{n} s + N \operatorname{cos.} \frac{4}{n} s$, &c. existente $Q = K + L + M + N +$, &c. ut sit Sr, sive $x = 1$, ubi planetæ Q et P incipiunt movere à lineâ conjunctionis SCD. Quantitates autem assumptæ K, L, M, &c. sunt exiguæ, ideoque erit $\frac{1}{x} = 1 + Q - K \operatorname{cos.} \frac{1}{n} s - L \operatorname{cos.} \frac{2}{n} s - M \operatorname{cos.} \frac{3}{n} s - N \operatorname{cos.} \frac{4}{n} s -$, &c. quamproximè.

proximè. Subſtituantur ergò in æquatione ſuprà traditâ valores quantitatum x, $\frac{1}{x}$, et U; et ſumptâ fluente, rejectis iis terminis qui ducuntur in altiorem quàm unam dimenſionem quantitatum ϕ, Q, K, L, &c. prodit $v = -\overline{\frac{2\phi khn}{t^3} - \frac{\phi}{t^3}A - Q} \times s$

$$+\overline{\frac{2\phi kn}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} + \frac{\phi}{t^3}B - K} \times n \times \text{ſin.}\ \frac{1}{n}s$$

$$+\overline{\frac{\phi kn}{t^3} \times \frac{S-V}{4} + \frac{\phi}{t^3} \times \frac{C}{2} - \frac{L}{2}} \times n \times \text{ſin.}\ \frac{2}{n}s$$

$$+\overline{\frac{\phi kn}{t^3} \times \frac{T-W}{9} + \frac{\phi}{t^3} \times \frac{D}{3} - \frac{M}{3}} \times n \times \text{ſin.}\ \frac{3}{n}s$$

$$+\overline{\frac{\phi kn}{t^3} \times \frac{V-X}{16} + \frac{\phi}{t^3} \times \frac{E}{4} - \frac{N}{4}} \times n \times \text{ſin.}\ \frac{4}{n}s +, \text{\&c.}$$

$+$ Z, deſignante Z quantitatem idoneam qua compleatur fluens. At, quoniam velocitas v ſupponitur nulla evadere, non ſolum ubi s, ſive arcus CQ $=$ o, id eſt, ubi planetæ verſantur in primâ illâ conjunctione, ſed etiam in omnibus aliis conjunctionibus ſubſequentibus, hoc eſt, ubi eſt angulus $\frac{1}{n}s$, ſeu PSQ $=$ o, vel $= r \times 180°$, ſcripto ſcilicet r pro quovis ex numeris naturalibus 1, 2, 3, 4, &c. fiet Z $=$ $\overline{\frac{2\phi khn}{t^3} - \frac{\phi}{t^3}A - Q} \times s$ adeoque

$$v = \overline{\frac{2\phi kn}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} + \frac{\phi}{t^3}B - K} \times n \times \text{ſin.}\ \frac{1}{n}s$$

$$+\overline{\frac{\phi kn}{t^3} \times \frac{S-V}{4} + \frac{\phi}{t^3} \times \frac{C}{2} - \frac{L}{2}} \times n \times \text{ſin.}\ \frac{2}{n}s$$

$$+\overline{\frac{\phi kn}{t^3} \times \frac{T-W}{9} + \frac{\phi}{t^3} \times \frac{D}{3} - \frac{M}{3}} \times n \times \text{ſin.}\ \frac{3}{n}s$$

$+$

$$+ \frac{\varphi k n}{t^3} \times \frac{V - X}{16} + \frac{\varphi}{t^3} \times \frac{E}{4} - \frac{N}{4} \times n \times \text{fin.}\, \frac{4}{n} s$$

$+$, &c.

Deinde, cùm fit tp, five $\dot{x}$ ad rp, five $x\dot{s}$, ut velocitas v qua defcribitur tp ad velocitatem $\frac{1}{x} - U$ qua defcribitur rp, erit $\dot{x} = v \times \frac{x\dot{s}}{\frac{1}{x} - U}$, five, quia valor velocitatis v componitur ex quantitatibus exiguis, $\dot{x} = v\dot{s}$ quamproximè, et $\frac{\dot{x}}{\dot{s}} = v$. Verùm etiam æquatio affumpta $x = 1 - Q + K \text{ cof. } \frac{1}{n} s + L \text{ cof. } \frac{2}{n} s + M \text{ cof. } \frac{3}{n} s +$, &c. dat $\frac{\dot{x}}{\dot{s}} = - K \times \frac{1}{n} \text{ fin. } \frac{1}{n} s - L \times \frac{2}{n} \text{ fin. } \frac{2}{n} s - M \times \frac{3}{n} \text{ fin. } \frac{3}{n} s - N \times \frac{4}{n} \text{ fin. } \frac{4}{n} s$, &c.

Habitis igitur duobus velocitatis v valoribus, eorum termini homologi ftatuantur æquales, atque inde obtinebuntur quantitates affumptæ, nempe

$$K = \frac{\varphi}{t^3} \times \frac{n^2}{n^2 - 1} \times \overline{2kR - \frac{2t^3}{k^2} \times \overline{n + \tfrac{1}{2}} - kT \times \overline{n - \tfrac{1}{2}} - S}$$

$$L = \frac{\varphi}{2t^3} \times \frac{n^2}{n^2 - 4} \times \overline{kS \times \overline{n + 1} - kV \times \overline{n - 1} - 2T}$$

$$M = \frac{\varphi}{3t^3} \times \frac{n^2}{n^2 - 9} \times \overline{kT \times \overline{n + \tfrac{3}{2}} - kW \times \overline{n - \tfrac{1}{2}} - 3V}$$

$$N = \frac{\varphi}{4t^3} \times \frac{n^2}{n^2 - 16} \times \overline{kV \times \overline{n + 2} - kX \times \overline{n - 2} - 4W}$$

&c.

indeque manifefta fit harum quantitatum progreffio: atque

atque hoc pacto habetur semper distantia x planetæ Q à Sole.

Jam ut definiatur planetæ Q motus verus qui designatur per s, dicatur w motus medius, sive, quod perinde est, tempus quo planeta descripserit arcum quemlibet Cr; atque ex demonstratis est $\dot{w} = \frac{x\dot{s}}{\frac{1}{x} - U}$; unde, substitutis valoribus quantitatum, x, $\frac{1}{x}$, et U, et sumptâ fluente, emergit

$$w = \overline{1 - 2Q + \frac{\varphi khn}{3}} \times s + \overline{2nK - \frac{\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}} \times \text{sin.}\ \frac{1}{n}s + \overline{nL - \frac{\varphi kn^2}{8t^3} \times \overline{S - V}} \times \text{sin.}\ \frac{2}{n}s + \overline{\frac{2nM}{3} - \frac{\varphi kn^2}{18t^3} \times \overline{T - W}} \times \text{sin.}\ \frac{3}{n}s + \overline{\frac{nN}{2} - \frac{\varphi kn^2}{32t^3} \times \overline{V - X}} \times \text{sin.}\ \frac{4}{n}s +, \&c. + Z$$

denotante Z quantitatem idoneam ut compleatur fluens. Sed, quia motus verus medio æqualis evadere supponitur in qualibet planetarum P et Q conjunctione cum Sole, id est, ubi angulus PSQ sive $\frac{1}{n}s$ æquatur, vel nihilo, vel angulo $r \times 180^\circ$, exhibente r quemvis ex numeris naturalibus 1, 2, 3, 4, &c. erit $Z = \overline{2Q - \frac{\varphi khn}{t^3}} \times s$. Ponantur igitur $F = -2nK + \frac{\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$, $G = -nL + \frac{\varphi kn^2}{8t^3} \times \overline{S - V}$, $H = -\frac{2nM}{3} + \frac{\varphi kn^2}{18t^3} \times \overline{T - W}$, $I = -\frac{nN}{2} + \frac{\varphi kn^2}{32t^3}$

×

$+ V - X$, &c. eritque motus verus, five $s = w + F$ fin. $\frac{1}{n}s + G \times$ fin. $\frac{2}{n}s + H \times$ fin. $\frac{3}{n}s + I \times$ fin. $\frac{1}{n}s +$, &c. vel, quia parum admodùm differt motus verus à motu medio $s = w + F \times$ fin. $\frac{1}{n}w + G \times$ fin. $\frac{2}{n}w + H \times$ fin. $\frac{3}{n}w + I \times$ fin. $\frac{4}{n}w +$, &c. Q. E. I.

COROLL. I.

His ita generatim definitis, ut specialis eliciatur in motu cujuspiam planetæ inæqualitatum mensura, determinandæ sunt quantitates assumptæ.

Itaque planeta P designet Terram, planeta Q Venerem, et quoniam est distantia Terræ ad distantiam Veneris à Sole ut 100000 ad 72333, hæc erit ratio k ad 1, adeoque $k = \frac{100000}{72333}$, $kk + 1 = tt = 2.91129$, $b = \frac{2k}{t^3} = 0.94975$; atque inde per methodum in Prop. I[a]. expositam prodibunt

R = 9.3925	V = 11.1964	Y = 5.3380
S = 16.6782	W = 8.8504	Z = 4.1029
T = 13.8877	X = 6.9045	&c.

Tum, existente periodo Terræ annuâ dierum 365.2565, et periodo Veneris dierum 224.701, est ex jam dictis $n = \frac{365.2565}{365.2565 - 224.701} = 2.59866$; et cùm gravitas in Solem sit juxta Newtonum ad gravitatem in Terram, paribus distantiis, ut 1 ad $\frac{1}{169282}$, erit $\varphi = \frac{1}{169282}$.

Unde, redactis in numeros formulis in hac propositione datis, emergunt

$K = 0.0000103$ $N = -0.0000065$
$L = 0.0000444$ $O = -0.0000024$
$M = 0.0000377$ $O' = -0.0000011$, &c.

Atque ex his tandem deducuntur

$F = -0.0000473$ $I = 0.0000100$
$G = -0.0001078$ $I' = 0.0000033$
$H = -0.0000684$ &c.

Hinc ergo habentur valores coefficientium æquationis $s = w + F \times \text{fin.} \frac{1}{n} w + G \times \text{fin.} \frac{2}{n} w + H \times \text{fin.} \frac{3}{n} w +$, &c. ubi s denotat motum Veneris verum, w motum medium, et $\frac{1}{n} w$ angulum PSQ five differentiam longitudinum heliocentricarum Terræ et Veneris; vel, reductis quantitatibus F, G, H, &c. ad exprimendas more aftronomico circuli partes, fit $s = w - 9''.76 \times \text{fin.} \frac{1}{n} w - 22''.24 \times \text{fin.} \frac{2}{n} w + 14''.11 \times \text{fin.} \frac{3}{n} w + 2''.06 \times \text{fin.} \frac{4}{n} w + 0''.68 \times \text{fin.} \frac{5}{n} w +$, &c.

Ut exemplum apponam, efto angulus PSQ five $\frac{1}{n} w = 40°$, ac prodibit $s = w - 15''.5$; motus igitur medius fuperat verum, eorumque differentia eft $15''.5$.

Computatâ hoc pacto differentiâ inter motum Veneris verum et medium refpectu Solis, fequenti modo innotefcet quanta evadat cùm e Terrâ fpectatur. Efto PSQ

PSQ (Fig. 4.) angulus exhibens, ut priùs, differentiam longitudinum planetarum P et Q tempore quovis dato, et in circulo RQ exhibente portionem orbitæ planetæ Q, sumatur arcus Qq æqualis differentiæ motuum prædictæ, et ductis Sq, Pq, centro P et radio Pq describatur arcus qr secans PQ in r, atque, ob parvitatem arcuum Qq, qr, erit $Qq : qr :: \text{rad.} : \text{sin. } PQq$;

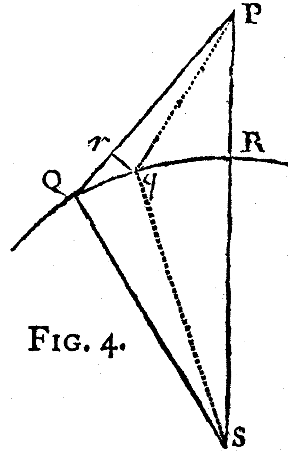

Fig. 4.

deinde $\frac{Qq}{QS} : \frac{qr}{PQ} :: \text{ang. } QSq : \text{ang. } QPq$; adeoque $\frac{\text{rad.}}{QS} : \frac{\text{sin. } PQq}{PQ} :: \text{ang. } QSq : \text{ang. } QPq$, unde $\text{ang. } QPq = \text{ang. } QSq \times \frac{QS}{PQ} \times \frac{\text{sin. } PQq}{\text{rad.}}$. Datis igitur angulo PSQ et distantiis PS, QS, dabitur distantia PQ, et angulus PQS, adeoque et angulus PQq: unde innotescet angulus quæsitus QPp, hoc est æquatio motûs, prout apparet spectatori in centro Terræ locato. Hincque, quamvis sit modica motûs Veneris inæqualitas telluris actione genita, qualis tamen sit ut pateat, libet eam in sequenti tabulâ oculis subjicere.

Hujus tabulæ columna prima exhibet angulum QPS, sive elongationem Veneris à Sole mediam; secunda indicat correctionem hujus elongationis, à conjunctione Veneris inferiore usque ad maximam ejus elongationem quæ in orbe circulari est 46° 19′ 50″ circiter. Tertia et quarta columna eodem modo exhibent elongationem Veneris, ejusque correctionem, à tempore elongationis maximæ usque ad conjunctionem superiorem.

 Elonga-

Elongatio Ven. à Sole.	Correctio.	Elongatio Ven. à Sole.	Correctio.
° ′ ″	″	° ′ ″	″
0	0	46 19 50	0
5	0	46	+ 2.3
10	0	45	5.1
15	0	40	9.5
20	— 0.5	35	7.3
25	0.8	30	1.8
30	1.5	25	— 4.4
35	2.8	20	9.2
40	2.9	15	11.2
45	2.7	10	10.2
46	1.7	5	6.0
46 19 50	0	0	0

Exempli gratiâ, ſi Venus à conjunctione inferiore digreſſa motu ſuo medio diſceſſerit à Sole angulo elongationis 40°, erit vera Veneris elongatio $40° - 2''.9 = 39° 59' 57''.1$: pariter, ſi ulteriùs delata Venus pervenerit ad eamdem elongationem 40°, erit tunc vera Veneris elongatio $40° 0' 9''.5$. Eædem omninò ſunt correctiones et cum iiſdem ſignis adhibendæ, ubi poſt conjunctionem ſuperiorem eædem eveniunt elongationes.

Coroll. II.

Ex præcedentibus etiàm deducitur diſtantia Veneris à Sole pro quolibet ejus cum Terrâ et Sole aſpectu, in hypothesi quod, seclusâ Terræ attractione, in orbitâ circulari revolveret. Sic, si angulus $\frac{1}{n}s$, seu PSQ sit 90°, vel 270°, æquatio $x = 1 - Q + K\ \text{cos.}\ \frac{1}{n}s + L\ \text{cos.}\ \frac{2}{n}s + M\ \text{cos.}\ \frac{3}{n}s + N\ \text{cos.}\ \frac{4}{n}s +$, &c. fit $x = 0.9999437$ circiter; et si sit PSQ = 180°, fit $x = 1.0000607$.

Unde, si distantia Veneris à Sole in conjunctione inferiore ponatur - - -	10000000
In quadraturis cum Terrâ erit ipsius distantia - - - - - - - - - -	9999437
In conjunctione superiore erit - - -	10000607

Item innotescit differentia inter tempus periodicum Veneris, quale nunc est, et tempus illud periodicum, quale foret, si unicâ Solis attractione in orbe circulari moveretur. Siquidem, cùm Venus post discessum suum à conjunctione ad eamdem redierit, æquatio generalis in propositione tradita, quæ exprimit relationem inter motum Veneris verum et medium, evadit

$$w = \overline{1 - 2Q + \frac{\varphi khn}{t^3}} \times s, \text{ sive } w = 1.0000066 \times s$$

circiter: unde tempus periodicum Veneris est ad tempus illud alterum periodicum, ut 1.0000066 ad 1; adeóque, si nulla foret gravitatio Veneris in Terram, revolutionem suam circa Solem minutis duobus horæ primis citiùs perageret.

PROPO-

Propositio III. Problema.

In ſyſtemate duorum planetarum in orbitis circularibus circa Solem revolventium, motum nodorum orbitæ planetæ interioris, quatenus ex vi planetæ exterioris oritur, inveſtigare.

Per motum nodorum hîc intelligendus eſt motus interſectionis plani orbis planetæ interioris cum plano orbis planetæ exterioris ſpectato ut immoto. Itaque eſto Sol in S (Fig. 5.) et centro S atque radio SQ de-

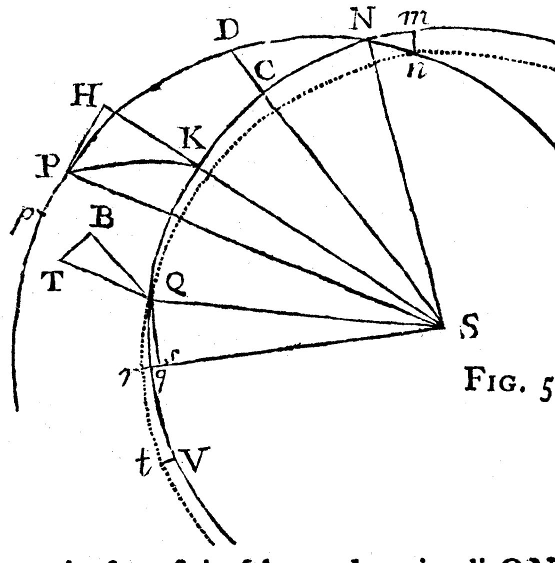

Fig. 5.

ſcribantur in ſuperficie ſphæræ duo circuli QN, PN, ſeſe interſecantes in N, quorum prior QN deſignet ſitum plani orbis planetæ interioris Q, et poſterior PN ſitum plani orbis planetæ exterioris, cujus locus ſit in rectâ SP productâ. Eodem centro S et radio SP deſcribatur circulus PK, cujus planum ſit plano

SQN

SQN perpendiculare, fecetque circulum QN in K, et in SK demittatur perpendiculum PH: tum ductâ QT parallelâ rectæ SP et TB in planum SQN normali, si linea QT exhibeat vim qua trahitur planeta Q in directione QT, seu SP, TB exhibebit vim qua distrahitur perpendiculariter à plano suæ orbitæ; eritque triangulum QTB simile triangulo SPH, atque adeò, TB : QT :: PH : SP :: sin. PK : 1; deinde in triangulo sphærico rectangulo PKN habetur, 1 : sin. PN :: sin. PNK : sin. PK; unde, conjunctis rationibus, et scripto c pro sinu anguli PNK ad radium 1, hoc est, pro sinu inclinationis orbis QN ad orbem PN, provenit $TB = QT \times c \times$ sin. PN. Sumatur jam arcus quàm minimus Qq, ad quem erigitur lineola perpendicularis qr, æqualis duplo spatio quod planeta Q percurrere posset impellente vi TB quo tempore in orbe suo describeret arcum illum Qq, et centro S descriptus circulus rQn secans circulum PN in n exhibebit situm orbis planetæ Q post tempus illud, nodo N translato in n; atque in QN demisso perpendiculo nm, et in Sq perpendiculo Qs, erit angulus qQr, sive NQn ad duplum angulum qQs, id est, ad angulum QSq, ut vis TB ad gravitatem (nempe 1) planetæ Q in Solem; hoc est, $\frac{nm}{\text{sin. } QN} : Qq :: TB : 1$; in triangulo autem rectangulo Nmn, est $Nn : nm :: 1 : c$; quarè conjunctis his rationibus, prodit $Nn = \frac{TB \times \text{sin. } QN \times Qq}{c}$; sed suprà invenimus $TB = QT \times c \times$ sin. PN, unde fit $Nn = QT \times$ sin. PN $\times$ sin. QN $\times Qq$.

Esto SC linea conjunctionis planetarum, fiatque, ut in propositione præcedente, arcus $CQ = s$, $Qq = \frac{\dot{s}}{SQ}$, SQ = 1; et, quia inclinatio orbis QN ad orbem PN exigua supponitur, erit etiam hîc ang. PSQ $= \frac{1}{n}s$ quamproximè; proindeque, posito arcu CN $= a$, erit QN $= s + a$ et PN $= s - \frac{1}{n}s + a$ quamproximè. Porrò, cùm lentissimè moveantur nodi, arcus CN spectari potest quasi invariabilis per multarum planetæ Q revolutionum seriem, atque adeò fluxio arcus QN eadem erit cum fluxione arcûs QC. His positis, habebitur sin. PN $\times$ sin. QN $= \frac{1}{2}$ cos. $\frac{1}{n}s - \frac{1}{2}$ cos. $\overline{2s - \frac{1}{n}s + 2a}$, estque per propositionem primam QT $= \frac{\varphi k}{z^3} - \frac{\varphi}{k^2} = \frac{\varphi k}{t^3}$ in R $- \frac{t^3}{k^3}$ + S cos. $\frac{1}{n}s$ + T cos. $\frac{2}{n}s$ + V cos. $\frac{3}{n}s$ + W cos. $\frac{4}{n}s$ +, &c. unde substitutis his valoribus in æquatione Nn = QT $\times$ sin. PN $\times$ sin. QN $\times$ Qq, et sumptâ fluente per methodum in Coroll. IV. lemmatis edoctam, prodibit summa omnium Nn, sive motus nodi, quo tempore planeta Q à loco conjunctionis C procedens in orbe suo descripserit arcum CQ, æqualis $\frac{\varphi k n}{2t^3}$ in

$$\frac{S}{2n}s + \overline{R - \frac{t^3}{k^3} + \frac{T}{2}} \times \text{sin.}\ \frac{1}{n}s + \frac{S+V}{4}\ \text{sin.}\ \frac{2}{n}s + \frac{T+W}{6}\ \text{sin.}\ \frac{3}{n}s +, \text{\&c.} + \frac{\varphi k n}{2t^3}\ \text{in Z} \times \text{sin.}\ 2a$$

$$- \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n-1}\ \text{sin.}\ \overline{2s - \frac{1}{n}s + 2a} - \frac{S}{2} \times \frac{1}{2n}\ \text{sin.}\ \overline{2s + 2a} - \frac{S}{2} \times \frac{1}{2n-2}\ \text{sin.}\ \overline{2s - \frac{2}{n}s + 2a} - \frac{T}{2}$$

$\times$

$\times \frac{1}{2n-3}$ fin. $\overline{2s - \frac{3}{n}s + 2a} - \frac{T}{2} \times \frac{1}{2n+1}$ fin. $\overline{2s + \frac{1}{n}s + 2a} - \frac{V}{2} \times \frac{1}{2n-4}$ fin. $\overline{2s - \frac{4}{n}s + 2a}$ $- \frac{V}{2} \times \frac{1}{2n+2}$ fin. $2s + \frac{2}{n}s + 2a$, &c. exiſtente $Z = \overline{2n-1}$ in $R - \frac{t^3}{k^3} \times \frac{1}{\overline{2n-1}|^2} + \frac{S}{\overline{2n-2+2n}}$ $+ \frac{T}{\overline{2n-3} \times \overline{2n+1}} + \frac{V}{\overline{2n-4} \times \overline{2n+2}} + \frac{W}{\overline{2n-5} \times \overline{2n+3}}$ $+$, &c. atque in his ſeriebus patet terminorum progreſſio. Q. E. I.

COROLL. I.

Hic liquet multas oriri in motu nodorum æquationes; ſed quia minutæ ſunt, et locum planetæ Q ferè nihil mutant, ideò ſatis erit rationem habere motûs nodorum medii et æquationis ſolîus *periodicæ*, qui ſic ex præcedentibus deducuntur. Cùm in planis parùm ad ſe inclinatis moveri ſupponantur planetæ P et Q, quoties revertentur ad conjunctionem, angulus PSQ, ſive $\frac{1}{n}s$, qui metitur eorum diſtantiam à ſe invicem, evadet $= 360°$ vel $= r \times 360°$, exiſtente r numero integro: et quia, ſumpto arcu quolibet A, eſt ſemper ſin. $\overline{r \times 360° + A} =$ ſin. A; hinc, ſi computatur motus nodi pro tempore conjunctionum, expreſſio illa generalis et prolixa in propoſitione tradita in hanc ſimplicem abit $\frac{\varphi k}{2t^3} \times \overline{\frac{S}{2}s - nZ \times \text{fin.}\ \overline{2s+2a} - \text{fin.}\ 2a}$, ſive per Coroll. I. lemmatis

$$\frac{\varphi k}{2t^3} \times \overline{\frac{S}{2}s - 2nZ \times \text{fin.}\ s \times \text{cof.}\ \overline{s + 2a}}.$$

 Hic

Hic eſt igitur motus nodorum factus, quo tempore planetæ P et Q à conjunctione provecti poſt quotlibetcunque revolutiones ad conjunctionem quamvis aliam pervenerint, exhibente s arcum à planetâ Q in ſuâ orbitâ intereà deſcriptum. Terminus $\frac{\phi k}{2t^3} \times 2nZ \times \text{ſin.}\, s \times \text{coſ.}\, \overline{s + 2a}$ indicat æquationem *periodicam* et facillimè computatur: cùmque hæc æquatio modò ſit additiva, modò ſubtractiva, patet termino altero $\frac{\phi k}{2t^3} \times \frac{S}{2} s$ exprimi generatìm motum nodi medium.

Coroll. II.

Eſto planeta P Terra, Q Venus, et revolutionem Veneris ab unâ conjunctione inferiore cum Terrâ ad alteram vocemus, brevitatis gratiâ, revolutionem *ſynodicam*; eritque poſt unam revolutionem ſynodicam $\frac{1}{n} s = 360°$, proindeque $s = n \times 360° = 935° \; 31'$; hic igitur eſt arcus deſcriptus à Venere inter duas ejuſdem generis conjunctiones. Hinc motus nodi medius tempore revolutionis unius ſynodicæ, qui juxta corollarium præcedens eſt $\frac{\phi k S}{4t^3} s$ fit $\frac{\phi k n S}{4t^3} = 360° = 23''.087$; atque hic motus imminutus in ratione temporis periodici Terræ circa Solem ad revolutionem Veneris ſynodicam, id eſt, in ratione 1 ad $n - 1$, evadit $14''.44$, motus ſcilicet annuus nodorum Veneris regreſſivus, qui ſpatio centum annorum fit $24' \; 4''$.

Æquatio periodica $\frac{\phi k n Z}{t^3} \times \text{ſin.}\, s \times \text{coſ.}\, \overline{s + 2a}$ ut adhuc ſimplicior evadat, ponamus arcum a ſive CN perexiguum

perexiguum esse vel nullum, id est, supponamus conjunctionem Terræ et Veneris fieri proximè in nodo, quemadmodum contingit hoc anno 1761, eritque æquatio periodica $\frac{\phi k n Z}{t^3} \times$ sin. $s \times$ cos. $s = \frac{\phi k n Z}{2 t^3}$ $\times$ sin. $2s$. Cùm igitur sit $Z = 32.33$ circiter, formula $\frac{\phi k S}{4 t^3} s - \frac{\phi k n Z}{2 t^3}$ sin. $2s$, quæ per corollarium præcedens exprimit generatìm motum nodi in qualibet serie revolutionum synodicarum confectum, fit $0.0000006855 \times s - 14''.2 \times$ sin. $2s$. Æquatio igitur periodica $14'.2 \times$ sin. $2s$, quam *generalem* voco, est ut sinus dupli arcûs à Venere descripti in datâ serie revolutionum synodicarum, nec ultra $14''.2$ ascendit. Jam, si pro s substituatur $935° \; 31'$, erit sin. $2s =$ sin. $71° \; 2'$, et regredientur nodi, in primâ revolutione synodicâ post conjunctionem factam in nodo, per arcum $23'' - 14''.2 \times$ sin. $71° \; 2' = 10''$: et, si r denotet numerum quemcumque revolutionum synodicarum, motus nodi, peractis illis revolutionibus, erit $r \times 23' - 14''.2 \times$ sin. $r \times 71° \; 2'$; pariterque, peractis revolutionibus quarum numerus est $r - 1$, idem motus erit $\overline{r - 1} \times 23'' - 14''.2 \times$ sin. $\overline{r - 1}$ $\times 71° \; 2'$; posterior motus ex priore auferatur, et remanebit $23'' - 14''.2 \times \overline{\text{sin.}\, r \times 71° \; 2' - \text{sin.}\, \overline{r-1} \times 71° \; 2'}$ $= 23'' - 14''.2 \times 2$ sin. $35° \; 31' \times$ cos. $\overline{r \times 71° \; 2' - 35° \; 31'}$ $= 23'' - 16''.5 \times$ cos. $\overline{2r - 1} \times 35° \; 31'$ pro motu nodi facto, tempore illius revolutionis synodicæ, cujus locum in serie revolutionum indicat numerus r. Exempli gratiâ, si desideretur motus nodi tempore revolutionis quartæ synodicæ post conjunctionem factam in nodo, erit $r = 4$, et regressus nodi erit

 23''

$23'' - 16''.5 \times \text{cos.}\ 7 \times 35^\circ\ 31 = 29''$. Sic ope hujus formulæ $23'' - 16''.5 \times \text{cos.}\ \overline{2r - 1} \times 35^\circ\ 31'$ facilè computatur sequens tabula, quæ exhibet regressum nodi Veneris in plano eclipticæ, pro duodecim sigillatim revolutionibus synodicis quæ proximè sequuntur conjunctionem Terræ et Veneris factam in nodo vel proximè ad nodum.

In revol. Ven. synod.	Regressus nodi Ven.	In revol. Ven. synod.	Regressus nodi Ven.
	″		″
1ª.	10	7ª.	26
2ª.	28	8ª.	39
3ª.	39	9ª.	30
4ª.	29	10ª.	11
5ª.	10	11ª.	8
6ª.	9	12ª.	25

Qui motus potest, cùm libuerit, ad annos communes reduci.

Denique patet æquationem periodicam, nempe $16''.5 \times \text{cos.}\ \overline{2r - 1} \times 35^\circ\ 31'$, quam *specialem* appello, ubi maxima est, evadere $16''\frac{1}{2}$; ac proinde regressum nodi in unâ revolutione synodicâ nusquam superare $39''\frac{1}{2}$, nec minorem esse $6''\frac{1}{2}$.

PROPO-

Propositio IV. Problema.

Iisdem positis, variationem inclinationis orbis planetæ interioris ad planum orbis planetæ exterioris determinare.

Esto NQV (Fig. 5.) quadrans circuli, cui erigatur perpendicularis Vt occurrens arcui nQr producto in t, eritque Vt mensura variationis inclinationis orbis NQV factæ quo tempore nodus N transfertur in n. Est autem Vt : nm :: sin. QV sive cos. QN : sin. QN, atque nm : Nn :: c : 1, c denotante sinum inclinationis orbis QN ad orbem PN, adeoque Vt : Nn :: c × cos. QN : sin. QN; unde Vt = Nn × $\frac{c \times \text{cos. QN}}{\text{sin. QN}}$, sive, quia per propositionem superiorem habetur Nn = QT × sin. PN × sin. QN × Qq, Vt = c × QT × sin. PN × cos. QN × Qq. Hinc, cum sit sin. PN × cos. QN = $\frac{1}{2}$ sin. $\overline{2s - \frac{1}{n}s + 2a}$ − $\frac{1}{2}$ sin. $\frac{1}{n}s$, sumptâ fluente prodit variatio inclinationis, quo tempore planeta Q à loco conjunctionis C movetur per arcum CQ, æqualis $-\frac{\phi c k n}{2t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$ × sin. vers. $\frac{1}{n}s$ + $\frac{S - V}{4}$ sin. vers. $\frac{2}{n}s$ + $\frac{T - W}{6}$ sin. vers. $\frac{3}{n}s$ + $\frac{V - X}{8}$ sin. vers. $\frac{4}{n}s$ +, &c. + $\frac{\phi c k n}{2t^3}$ in − Z × sin. vers. $2a$ + $\overline{R - \frac{t^3}{k^3}}$ × $\frac{1}{2n - 1}$ sin. vers. $\overline{2s - \frac{1}{n}s + 2a}$ + $\frac{S}{2} \times \frac{1}{2n}$ sin. vers. $\overline{2s + 2a}$ + $\frac{S}{2} \times \frac{1}{2n - 2}$ sin. vers.

2 s

$$\overline{2s - \tfrac{2}{n}s + 2a} + \tfrac{T}{2} \times \tfrac{1}{2n+1}\ \text{ſin. verſ.}\ \overline{2s + \tfrac{1}{n}s + 2a}$$
$$+ \tfrac{T}{2} \times \tfrac{1}{2n-3}\ \text{ſin. verſ.}\ \overline{2s - \tfrac{3}{n}s + 2a} + \tfrac{V}{2} \times \tfrac{1}{2n+2}$$
$$\text{ſin. verſ.}\ \overline{2s + \tfrac{2}{n}s + 2a} + \tfrac{V}{2} \times \tfrac{1}{2n-4}\ \text{ſin. verſ.}$$
$$\overline{2s - \tfrac{4}{n}s + 2a} + \tfrac{W}{2} \times \tfrac{1}{2n+3}\ \text{ſin. verſ.}\ \overline{2s + \tfrac{3}{n}s + 2a}$$
$$+ \tfrac{W}{2} \times \tfrac{1}{2n-5}\ \text{ſin. verſ.}\ \overline{2s - \tfrac{5}{n}s + 2a},\ \text{\&c.}$$

Exiſtente hîc eodem valore quantitatis Z ac in propoſitione præcedente. Q. E. I.

Coroll.

Si computetur variatio inclinationis pro tempore conjunctionum, facilè obtinebitur; hæc enim per formulam in propoſitione traditam evadit $\frac{\varphi c k n}{2 t^3} \times Z \times \overline{\text{ſin. verſ.}\ \overline{2s + 2a} - \text{ſin. verſ.}\ 2a}$ quæ itèm, ſi prima conjunctionum, à qua ſumitur motûs exordium, ſtatuatur in nodo, fit $\frac{\varphi c k n}{1 t^3} \times Z \times$ ſin. verſ. $2s$.

Hoc eſt igitur decrementum inclinationis orbis planetæ Q factum in qualibet ſerie revolutionum ad conjunctionem, deſignante s arcum intereà à planetâ circa Solem deſcriptum. Conferatur hæc inclinationis variatio cum æquatione nodi periodicâ eodem tempore genitâ, prout in propoſitione ſuperiore definitur, et patebit priorem eſſe ad poſteriorem ut $c \times$ ſin. verſ. $2s$ ad ſin. $2s$.

Ut ad orbem Veneris hæc transferantur, quem ſi inclinari ad orbem Terræ ſupponatur angulo $3° 23' 20''$, erit

erit $\frac{\phi c k n}{2t^3} \times Z \times$ ſin. verſ. $2s = 0''.84 \times$ ſin. verſ. $2s$. Unde palàm fit: 1°. in quacumque ſerie revolutionum ſynodicarum, poſt conjunctionem factam in nodo, decrementum inclinationis orbitæ Veneris ad eclipticam non ſuperare $2 \times 0''.84 = 1''.68$, quod è Terrâ ſpectatum evadit $4''.4$: 2°. cùm, peractâ unâ revolutione ſynodicâ, ſit ſin. verſ. $2s =$ ſin verſ. $71° 2'$, inclinationis decrementum pro qualibet ſerie revolutionum ſynodicarum quarum numerus eſt r, eſſe $0''.84 \times$ ſin. verſ. $r \times 71° 2'$, et pro ſerie revolutionum quarum numerus eſt $r - 1$, eſſe $0''.84 \times$ ſin. verſ. $\overline{r - 1 \times 71° 2'}$; unde horum decrementorum differentia $0''.84 \times \overline{\text{ſin. verſ. } r \times 71° 2' - \text{ſin. verſ. } \overline{r - 1} \times 71° 2}$ $= 0''.84 \times 2$ ſin. $\underline{35° 31'} \times$ ſin. $\overline{2r - 1} \times 35° 31' =$ $0''.98 \times$ ſin. $\overline{2r - 1} \times 35° 31'$, exprimit variationem inclinationis genitam tempore revolutionis ſynodicæ illius, cujus locum in ſerie revolutionum denotat numerus r: atque hæc variatio, ut patet, nuſquam excedit $0''.98$ è Sole conſpecta, quæ ſpectatori in centro Terræ collocato ſub angulo $2''\frac{1}{2}$ apparebit. Cum igitur tantilla ſit orbitæ Veneris inclinationis variatio, non videtur operæ pretium de eâ ulteriùs exquirere.

Demonſtratis, quæ ad perturbationem motûs planetæ interioris ſpectant, ſupereſt ut, quibus perturbationibus afficiatur motus planetæ exterioris, viciſſim expendamus.

Propositio V. Problema.

In ſyſtemate duorum planetarum circa Solem in orbibus penè circularibus revolventium, determinare vim planetæ interioris ad perturbandum motum exterioris.

Simili ratiocinio ei, quod in propoſitione primâ uſurpavimus, etiam hoc problema ſolvitur. Itaque poſitâ unitate pro diſtantiâ planetæ P à Sole, ubi ambo planetæ P et Q conjunguntur cum Sole, (Fig. 1.) fiat $SP = x$, $SQ = k$, $PQ = z$. Sit 1 ad ϕ ut gravitatio planetæ P in Solem in diſtantiâ 1 ad ejuſdem planetæ P gravitationem in planetam Q in eâdem diſtantiâ, eritque $\frac{\phi}{z^2}$ gravitas planetæ P in planetam Q in diſtantiâ PQ. Productâ, ſi opus eſt, PQ ad O ut ſit $PO = \frac{\phi}{z^2}$, et ductâ OI parallelâ rectæ QS occurrente PS productæ in I, reſolvatur vis PO in vires PI et OI, eritque propter ſimilia triangula PQS, POI, vis $OI = \frac{PO \times QS}{PQ} = \frac{\phi k}{z^3}$, atque vis $PI = \frac{PO \times PS}{PQ} = \frac{\phi x}{z^3}$ ſive vis $PI = \frac{\phi}{z^3}$ quamproximè. Vis OI impellit planetam P in directione parallelâ rectæ SQ, et in eundem ſenſum urgetur Sol vi $\frac{\phi}{k^2}$ qua gravitat in planetam Q: exceſſu igitur ſolo vis prioris ſupra poſteriorem, nempe $\frac{\phi k}{z^3} - \frac{\phi}{k^2}$, cenſendus eſt urgeri planeta P in directione parallelâ rectæ SQ.

Porrò

Porrò vis $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$ ea pars, quæ agit perpendiculariter ad radium PS, eſt $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PSQ, atque altera pars, quæ amovet planetam P à Sole ſecundum PS, eſt $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ coſ. PSQ. Auferatur hæc poſterior vis ex vi PI, et manebit vis $\frac{\varphi}{z^3} + \overline{\frac{\varphi}{k^2} - \frac{\varphi k}{z^3}} \times$ coſ. PSQ, qua planeta P urgetur verſus Solem.

Eſto DCS (Fig. 2.) linea conjunctionis planetarum, et arcus DP, ſive angulus DSP vocetur s, denotentque P et Q reſpectivè tempora periodica planetarum P et Q, eritque, poſito $n = \frac{Q}{P - Q}$, ang. PSQ $= \frac{1}{n}s$. Tum, ſi fiat $t^2 = 1 + kk$, et $b = \frac{2k}{t^2}$, erit uti in Prop. I. expoſuimus, $z^2 = t^2 \times \overline{1 - b \text{ coſ. } \frac{1}{n}s}$, atque $\frac{1}{z^3} = \frac{1}{t^3} \times \overline{R + S \text{ coſ. } \frac{1}{n}s + T \text{ coſ. } \frac{2}{n}s + V \times \text{coſ. } \frac{3}{n}s +, \&c.}$ et quemadmodùm ibi erat $b = \frac{2 PS \times SQ}{\overline{PS}|^2 + \overline{SQ}|^2}$, hîc item eſt $b = \frac{2 PS \times SQ}{\overline{PS}|^2 + \overline{SQ}|^2}$, adeoque valores quantitatum aſſumptarum R, S, T, &c. iidem hîc ſunt ac in propoſitione primâ.

Unde vis $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PSQ, qua ſollicitatur planeta P in directione ad radium PS perpendiculari, ſic exprimetur $\frac{\varphi k}{t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times$ ſin. $\frac{1}{n}s + \frac{S - V}{2}$

 ſin.

$\text{fin.}\,\frac{2}{n}s + \frac{T - W}{2}\,\text{fin.}\,\frac{3}{n}s + \frac{V - X}{2}\,\text{fin.}\,\frac{4}{n}s +$, &c.

Et vis $\frac{\varphi}{z^3} + \overline{\frac{\varphi}{k^2} - \frac{\varphi k}{z^3}} \times \text{cof. PSQ}$, qua urgetur planeta P in Solem fecundum radium PS, fiet $\frac{\varphi}{t^3}$ in $R - \frac{kS}{2} - \overline{kR + \frac{kT}{2} - \frac{t^3}{k^2} - S} \times \text{cof.}\,\frac{1}{n}s$ $- \frac{kS + kV - 2T}{2}\,\text{cof.}\,\frac{2}{n}s - \frac{kT + kW - 2V}{2}\,\text{cof.}\,\frac{3}{n}s$ $- \frac{kV + kX - 2W}{2}\,\text{cof.}\,\frac{4}{n}s +$, &c. Q. E. I.

Propositio VI. Problema.

Inæqualitates motûs planetæ exterioris ex viribus prædictis ortas inveftigare.

Per analyfim in propofitione fecundâ inftitutam vis ad radium PS perpendicularis generabit accelerationem, vel retardationem velocitatis, dum arcus quilibet DP defcribitur à planeta P, æqualem $\frac{\varphi k n}{t^3}$ in $b - \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{cof.}\,\frac{1}{n}s - \frac{S - V}{4}\,\text{cof.}\,\frac{2}{n}s$ $- \frac{T - W}{6}\,\text{cof.}\,\frac{3}{n}s - \frac{V - X}{8}\,\text{cof.}\,\frac{4}{n}s -$, &c. $=$ U exiftente $b = R - \frac{t^3}{k^3} - \frac{T}{2} + \frac{S - V}{4} + \frac{T - W}{6}$ $+ \frac{V - X}{8} +$, &c.

Deinde fi fcribatur p pro vi illâ planetæ Q qua urgetur planeta P in Solem, prout in propofitione præcedente definita eft, et v pro velocitate afcenfûs vel defcenfûs planetæ P fecundum radium PS, et jam fupponatur ſupponatur $SP = x = 1 - Q + K \text{ coſ. } \frac{1}{n}s + L$ $\text{coſ. } \frac{2}{n}s + M \text{ coſ. } \frac{3}{n}s + N \text{ coſ. } \frac{4}{n}s +$, &c. exiſtente $Q = K + L + M + N +$, &c. erit $\frac{1}{x^2} + p$ vis centripeta planetæ P, et $\frac{1}{x} \times \overline{\frac{1}{x} + U}|^2$ ejuſdem vis centrifuga, atque inde habebitur $\dot{v} =$

$$\overline{\frac{1}{x^2} + p - \frac{1}{x} \times \overline{\frac{1}{x} - U}|^2} \times \frac{x\dot{s}}{\frac{1}{x} + U}.$$

Tum reſtitutis valoribus quantitatum U, p, x, et proſequendo calculum prout in Prop. II. poſitis

$$A = Kn + \frac{2\varphi kn^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}} - \frac{\varphi n}{t^3} \times \overline{kR - \frac{t^3}{k^2} - S + \frac{kT}{2}}$$

$$B = L \times \frac{n}{2} + \frac{\varphi kn^2}{4t^3} \times \overline{S - V} - \frac{\varphi n}{4t^3} \times \overline{kS + kV - 2T}$$

$$C = M \times \frac{n}{3} + \frac{\varphi kn^2}{9t^3} \times \overline{T - W} - \frac{\varphi n}{6t^3} \times \overline{kT + kW - 2V}$$

$$D = N \times \frac{n}{4} + \frac{\varphi kn^2}{16t^3} \times \overline{V - X} - \frac{\varphi n}{8t^3} \times \overline{kV + kX - 2W}$$

&c.

prodibit $v = \frac{\varphi}{t^3} \times \overline{R - \frac{kS}{2} - \frac{2\varphi khn}{t^3} - Q} \times s$ $+ A \times \text{ſin. } \frac{1}{n}s + B \times \text{ſin. } \frac{2}{n}s + C \times \text{ſin. } \frac{3}{n}s + D$ $\times \text{ſin. } \frac{4}{n}s +$, &c. $+ Z$, et factâ hypotheſi quòd ſit $v = 0$ ubi angulus $PSQ = 0$, vel $r \times 180°$, exprimente r unum ex numeris naturalibus 1, 2, 3, 4, &c. erit $Z = -\frac{\varphi}{t^3} \times \overline{R - \frac{kS}{2} - \frac{2\varphi khn}{t^3} - Q} \times s$,

 ac

ac proinde $v = A \times \text{fin.}\, \frac{1}{n}s + B \times \text{fin.}\, \frac{2}{n}s + C \times \text{fin.}\, \frac{3}{n}s + D \times \frac{4}{n}s +$, &c.

Tùm, quia vis centripeta hîc excedere fupponitur vim centrifugam, cùm contrarium fuppofitum fuerit in propofitione fecundâ, habetur $-\dot{x} = v \times \frac{x\dot{s}}{\frac{1}{x} + U}$ five $-\dot{x} = v\dot{s}$ proximè, et $-\frac{\dot{x}}{\dot{s}} = v = K \times \frac{1}{n}\, \text{fin.}\, \frac{1}{n}s + L \times \frac{2}{n}\, \text{fin.}\, \frac{2}{n}s + M \times \frac{3}{n}\, \text{fin.}\, \frac{3}{n}s + N \times \frac{4}{n}\, \text{fin.}\, \frac{4}{n}s +$, &c.

Unde factâ collatione terminorum hujus valoris velocitatis v cum terminis homologis valoris fupra inventi, emergent

$$K = -\frac{\varphi}{t^3} \times \frac{n^2}{n^2 - 1} \times \overline{2kR - \frac{2t^3}{k^2} \times \overline{n - \tfrac{1}{2}} - kT \times \overline{n + \tfrac{1}{2}} + S}$$

$$L = -\frac{\varphi}{2t^3} \times \frac{n^2}{n^2 - 4} \times \overline{kS \times \overline{n - 1} - kV \times \overline{n + 1} + 2T}$$

$$M = -\frac{\varphi}{3t^3} \times \frac{n^2}{n^2 - 9} \times \overline{kT \times \overline{n - \tfrac{3}{2}} - kW \times \overline{n + \tfrac{3}{2}} + 3V}$$

$$N = -\frac{\varphi}{4t^3} \times \frac{n^2}{n^2 - 16} \times \overline{kV \times \overline{n - 2} - kX \times \overline{n + 2} + 4W}$$

&c.

atque ità patet hujufmodi quantitatum progreffio. Innotefcet igitur x, feu diftantia planetæ P à Sole in quovis ejus cum planetâ Q afpectu.

Ut obtineatur planetæ P motus verus s, defignet w motum medium, et cùm fit $\dot{w} = \frac{x\dot{s}}{\frac{1}{x} + U}$, fubfti-

tuantur

:uantur valores quantitatum x, U, et ſumptâ fluente, poſitis

$$F = 2nK + \frac{\phi k n^2}{t^3} \times \overline{R - \frac{t^3}{k^3} - \frac{T}{2}}$$

$$G = nL + \frac{\phi k n^2}{8t^3} \times \overline{S - V}$$

$$H = \frac{2nM}{3} + \frac{\phi k n^2}{18t^3} \times \overline{T - W}$$

$$I = \frac{nN}{2} + \frac{\phi k n^2}{32t^3} \times \overline{V - X}$$

&c.

proveniet $w = \overline{1 - 2Q - \frac{\phi k h n}{t^3}} \times s + F \times \text{ſin.}\ \frac{1}{n}s + G \times \text{ſin.}\ \frac{2}{n}s + H \times \text{ſin.}\ \frac{3}{n}s + I \times \text{ſin.}\ \frac{4}{n}s +$, &c. $+ Z$.

Et factâ hypotheſi quod motus verus coincidat cum medio ubi eſt $\frac{1}{n}s$, ſeu angulus PSQ $= 0$, vel $= r \times 180°$, exhibente r quemvis ex numeris 1, 2, 3, 4, &c. erit $Z = \overline{2Q + \frac{\phi k h n}{t^3}} \times s$; ac proinde, ſcriptis $\frac{1}{n}w$, $\frac{2}{n}w$, &c. pro $\frac{1}{n}s$, $\frac{2}{n}s$, &c. quia parùm admodùm differt motus verus à medio, habetur motus verus, ſive $s = w - F \times \text{ſin.}\ \frac{1}{n}w - G \times \text{ſin.}\ \frac{2}{n}w - H \times \text{ſin.}\ \frac{3}{n}w - I \times \text{ſin.}\ \frac{4}{n}w -$, &c. Q. E. I.

COROLL. I.

Deſignet jam planeta P Terram, Q Venerem, et quia poſuimus eſſe diſtantiam mediocrem Terræ à Sole

Sole ad diſtantiam mediocrem Veneris à Sole ut 1 ad k, erit hîc $k = 0.72333$, atque $t = \sqrt{1 + kk} = 1.234182$. Item eſt $n = \frac{Q}{P - Q} = \frac{224.701}{365.2565 - 224.701} = 1.59866$. Quantitates b, R, S, T, &c. eoſdem hîc retinent valores quos habebant in Coroll. I. Prop. II. Verùm, ut motuum Terreſtrium accurata inſtitueretur computatio, dignoſcere neceſſe eſſet effectus aliquos ab actione Veneris provenientes, ex quibus derivare liceret vim attractivam iſtius planetæ, ſed quia ſpeciales hujuſmodi effectus nulli, quantùm noverimus, obſervationibus aſtronomicis explorati habentur, propterea vim Veneris nunc conjecturâ definiemus, ut inde inæqualitates in motu Telluris computatæ, atque cum obſervationibus aſtronomicis collatæ inſervire poſthac poſſint ad eamdem vim certiùs determinandam. Itaque ſupponemus gravitatem in Solem eſſe ad gravitatem in Venerem, paribus diſtantiis, ut 400000 ad 1, hoc eſt, eſſe $\phi = \frac{1}{400000}$. Qui tamen valor vis ϕ ſi major vel minor poſteà deprehenſus fuerit, in eâdem ratione ſequentes omnes determinationes augendæ ſunt, vel minuendæ, adeoque ad juſtam menſuram facillimè reducentur. Erunt igitur

$K = -0.00000575$	$N = 0.00000090$
$L = 0.00001643$	$O = 0.00000039$
$M = 0.00000259$	$O' = 0.00000022$, &c.

Indeque colliguntur

$F = -0.00002459$	$I = 0.00000105$
$G = 0.00002795$	$I' = 0.00000042$
$H = 0.00000345$	&c.

atque reductis quantitatibus F, G, H, &c. in partes circuli,

circuli, tandem habetur $s = w + 5''.07 \times \text{fin.}\ \frac{1}{n} w$ $- 5''.76 \times \text{fin.}\ \frac{2}{n} w - 0''.71 \times \text{fin.}\ \frac{3}{n} w - 0''.22$ $\times \text{fin}\ \frac{4}{n} w -$, &c. ubi s denotat motum Terræ verum, w motum medium, et $\frac{1}{n} w$ angulum PSQ, five differentiam longitudinum heliocentricarum Terræ et Veneris.

Inde computatur fequens tabula exhibens æquationem motûs Solis pro variâ diftantiâ Veneris à Terrâ quam metitur angulus PSQ, five pro variâ differentiâ longitudinum heliocentricarum Terræ et Veneris quam metitur arcus circuli maximi inter Terram et Venerem interjectus et fecundum feriem fignorum à loco Terræ computatus.

Diff.

Diff. long. hel. Terræ et Ven.	Æquatio motûs Solis.	Diff. long. hel. Terræ et Ven.	Æquatio motûs Solis.
°	″	°	″
Sig. 0. 0	— 0	Sig. VI. 0	— 0
10	1.6	10	2.6
20	2.8	20	5.0
30	3.4	30	7.0
Sig. I. 10	3.1	Sig. VII. 10	8.4
20	2.1	20	9.1
30	0.4	30	9.2
Sig. II. 10	+ 1.6	Sig. VIII. 10	8.6
20	3.8	20	7.5
30	5.8	30	5.8
Sig. III. 10	7.5	Sig. IX. 10	3.8
20	8 6	20	1.6
30	9.2	30	+ 0.4
Sig. IV. 10	9.1	Sig. X. 10	2.1
20	8.4	20	3.1
30	7.0	30	3.4
Sig. V. 10	5.0	Sig. XI. 10	2.8
20	2.6	20	0.6
30	0.	30	0

COROLL. II.

Si tellus gravitate suâ in Solem in circulo revolvi posse supponatur, adveniente Veneris actione variari debere distantiam ejus à Sole patet ex hac propositione. Esto angulus $\frac{1}{n}s$, seu PSQ = 90°, vel 270°, atque æquatio generalis $x = 1 - Q + K \text{ cos. } \frac{1}{n}s + L \text{ cos. } \frac{2}{n}s + M \text{ cos. } \frac{3}{n}s +$, &c. in hanc abit $x = 0.9999693$; et si sit PSQ = 180°, fit $x = 1.0000053$.

Unde si distantia Terræ à Sole, ubi versatur in conjunctione cum Venere, ponatur - - - - - - - - - - -	10000000
In quadraturis cum Venere erit ipsius distantia - - - - - - - - - - - -	9999693
Atque in oppositione - - - - - - -	10000053

PROPOSITIO VII. PROBLEMA.

In systemate duorum planetarum in circulis circa Solem revolventium, motum nodorum orbis planetæ exterioris in plano orbis planetæ interioris investigare.

Esto P locus planetæ exterioris (Fig. 5.) in orbe suo PN, SQ recta conjungens Solem et planetam interiorem, et dicatur c sinus inclinationis duorum orbium ad se invicem ad radium 1, atque per propositionem quintam est $\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}$ vis qua planeta P amovetur ab orbe suo secundum directionem parallelam rectæ SQ, hujusque vis ea pars quæ perpendiculariter agit in planum orbis PN, per ſimile ratiocinium quo uſi ſumus in Prop. III. prodit æqualis $c \times$ ſin. QN $\times \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}}$, et motus interſectionis plani orbis PN cum plano orbis QN fit $\overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times$ ſin. PN $\times$ ſin. QN $\times$ P$\dot{p}$ quo tempore planeta P deſcribit in orbe ſuo arcum quàm minimum Pp.

Deinde ſi deſignaverit D locum planetæ P ubi verſatur in conjunctione cum planetâ interiore, et ponantur DP $= s$, P$p = \dot{s}$, DN $= a$, erit PN $= s + a$, QN $= s + \frac{1}{n}s + a$ quamproximè, atque ſin. PN $\times$ ſin. QN $= \frac{1}{2}$ coſ. $\frac{1}{n}s - \frac{1}{2}$ coſ. $\overline{2s + \frac{1}{n}s + 2a}$.

Unde, calculum proſequendo uti in propoſitione tertiâ, motus nodorum factus, quo tempore planeta P à loco conjunctionis D diſcedens deſcripſerit in orbe ſuo arcum quemlibet DP, exprimetur per

$$\frac{\varphi k n}{2t^3} \text{ in } \frac{S}{2n}s + \overline{R - \frac{t^3}{k^3} + \frac{T}{2}} \times \text{ſin.} \frac{1}{n}s + \frac{S+V}{4} \text{ſin.} \frac{2}{n}s$$

$$+ \frac{T+W}{6} \text{ſin.} \frac{3}{n}s + \frac{V+X}{8} \text{ſin.} \frac{4}{n}s +, \text{ \&c.}$$

$$+ \frac{\varphi k n}{2t^3} \text{ in } Z \times \text{ſin.}\, 2a - \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1} \text{ſin.} \overline{2s + \frac{1}{n}s + 2a}$$

$$- \frac{S}{2} \times \frac{1}{2n} \text{ſin.} \overline{2s + 2a} - \frac{S}{2} \times \frac{1}{2n+2} \text{ſin.} \overline{2s + \frac{2}{n}s + 2a}$$

$$- \frac{T}{2} \times \frac{1}{2n-1} \text{ſin.} \overline{2s - \frac{1}{n}s + 2a} - \frac{T}{2} \times \frac{1}{2n+3} \text{ſin.} \overline{2s + \frac{3}{n}s + 2a} - \frac{V}{2} \times \frac{1}{2n-2} \text{ſin.} \overline{2s - \frac{2}{n}s + 2a}$$

—

$-\frac{V}{2}\times\frac{1}{2n+4}$ fin. $\overline{2s+\frac{4}{n}s+2a}-\frac{W}{2}\times\frac{1}{2n-3}$ fin. $\overline{2s-\frac{3}{n}s+2a}-\frac{W}{2}\times\frac{1}{2n+5}$ fin. $\overline{2s+\frac{5}{n}s+2a}$, &c. existente $Z = 2n+1$ in $R-\frac{t^3}{k}\times\frac{1}{\overline{2n+1}|^2}+\frac{S}{\overline{2n}\times\overline{2n+2}}+\frac{T}{\overline{2n-1}\times\overline{2n+3}}+\frac{V}{\overline{2n-2}\times\overline{2n+4}}+\frac{W}{\overline{2n-3}\times\overline{2n+5}}$ $+$, &c. In quibus feriebus manifefta eft terminorum progreffio. Q. E. I.

Coroll.

Hinc in conjunctionibus expreffio motûs nodi evadit $\frac{\varphi k}{2t^3}\times\overline{\frac{S}{2}s-nZ\times\text{fin.}\ \overline{2s+2a}-\text{fin.}\ 2a}$. Hicque eft motus nodi factus quo tempore planetæ P et Q à conjunctione procedentes ad conjunctionem quamvis aliam pervenerint, exhibente s arcum à planetâ P in fuâ orbitâ intereà defcriptum. Terminus $\frac{\varphi k}{2t^3}\times\frac{S}{2}s$ exprimit motum nodi medium, et terminus alter $\frac{\varphi kn}{2t^3}Z\times\overline{\text{fin.}\ \overline{2s+2a}-\text{fin.}\ 2a}$ indicat æquationem *periodicam generalem*; vel etiam, fi conjunctio illa à qua defumitur computationis initium, fieri fupponatur in nodo, vel propè ad nodum, æquatio periodica generalis fit $\frac{\varphi kn}{2t^3}Z\times$ fin $2s$.

Defignet jam planeta P Terram, Q Venerem, eritque poft unam revolutionem fynodicam, id eft, poft revolutionem Veneris ad Terram, $\frac{1}{n}s = 360°$,

 proindeque

proindeque $s = n \times 360° = 575° \; 31'$. Quare motus nodi medius huic temporis ſpatio congruens fit $\frac{\phi k n}{4t^3} S \times 360°$, qui imminutus in ratione revolutionis Terræ circa Solem ad ejuſdem revolutionem ad Venerem, hoc eſt, in ratione 1 ad n, evadit $\frac{\phi k}{4t^3} S \times 360°$ $= 5''.20$, motus ſcilicet nodi medius annuus quo regreditur interſectio planorum orbium Terræ ac Veneris; atque hic motus ſpatio centum annorum fit $8' \; 40''$.

In computo æquationis *periodicæ generalis* $\frac{\phi k n}{2t^3} Z \times$ ſin. $2s$, advertendum eſt omnes terminos, ex quibus componitur valor quantitatis Z, eoſdem hîc eſſe ac in Prop. III. præter terminum primum $\overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1}$ qui ob diverſum valorem quantitatum t et k diverſus eſt. Hîc igitur provenit $Z = 31.59$, adeoque $\frac{\phi k n}{2t^3} Z \times$ ſin. $2s = 5'' \times$ ſin. $2s$; unde patet æquationem hanc nunquam ſuperare $5''$. Motus igitur nodi verus, nimirùm $\frac{\phi k}{2t^3} \times \overline{\frac{S}{2} s - nZ \times \text{ſin. } 2s}$, peractâ unâ revolutione ſynodicâ poſt conjunctionem factam in nodo, evadit $8''.3 - 5'' \times$ ſin. $71°. \; 2'$, quia tunc eſt ſin. $2s =$ ſin. $2 \times 575°. \; 31' =$ ſin. $71°. \; 2'$; et per ratiocinium ſimile ei, quod in Coroll. II. Prop. III. uſurpatum eſt, conſtabit $8''.3 - 5''.8 \times$ coſ. $\overline{2r - 1} \times 35°. \; 31'$ exprimere regreſſum nodi factum tempore illius revolutionis ſynodicæ, cujus lo-

5 cum

cum in ſerie revolutionum indicat numerus *r*. Hinc computatur tabula ſequens quæ exhibet regreſſum nodi orbitæ Terreſtris in plano orbis Veneris pro duodecim ſigillatìm revolutionibus ſynodicis quæ proximè ſequuntur conjunctionem Terræ et Veneris factam in nodo, vel proximè ad nodum.

In revol. ſynod.	Regreſſus nodi Ter.	In revol. ſynod.	Regreſſus nodi Ter.
	$''$		$''$
1	4	7	9
2	10	8	14
3	14	9	11
4	10	10	4
5	4	11	3
6	3	12	9

Patet autem æquationem *periodicam ſpecialem*, nempe $5''.8 \times \text{coſ.}\ \overline{2r - 1} \times 35^\circ.\ 31'$, ubi maxima eſt, evadere $5''.8$, et regreſſum nodi in quavis revolutione Terræ ad Venerem non aſſurgere ultra $14''$, nec minui citra $2''\frac{1}{2}$.

Propositio VIII. Problema.

Iiſdem poſitis, variationem inclinationis orbis planetæ exterioris ad planum orbis planetæ interioris determinare.

Deſignet I variationem inclinationis factam quo tempore planeta P deſcribit arcum quàm minimum

Pp,

Pp, et N motum nodi eodem tempore confectum, ac per ratiocinium omnino simile ei quod adhibitum est in propositione quartâ habetur $I = N \times \frac{c \times \text{cos. } PN}{\text{sin. } PN}$: sed per propositionem præcedentem est $N = \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times \text{sin. } PN \times \text{sin. } QN \times Pp$, adeoque fit $I = \overline{\frac{\varphi k}{z^3} - \frac{\varphi}{k^2}} \times c \times \text{cos. } PN \times \text{sin. } QN \times Pp$.

Unde, cùm hîc sit $PN = s + a$, $QN = s + \frac{1}{n}s + a$, proindeque $\text{cos. } PN \times \text{sin. } QN = \frac{1}{2} \text{ sin. } \frac{1}{n}s + \frac{1}{2} \text{ sin. } \overline{2s + \frac{1}{n}s + 2a}$, sumptâ fluente prodit variatio inclinationis genita, quo tempore planeta descripserit in orbe suo arcum quemlibet DP à loco conjunctionis D, æqualis $\frac{\varphi c k n}{2t^3}$ in $\overline{R - \frac{t^3}{k^3} - \frac{T}{2}} \times \text{sin. vers. } \frac{1}{n}s + \frac{S - V}{4} \text{ sin. vers. } \frac{2}{n}s + \frac{T - W}{6} \text{ sin. vers. } \frac{3}{n}s + \frac{V - X}{8} \text{ sin. vers. } \frac{4}{n}s +$, &c. $+ \frac{\varphi c k n}{2t^3}$ in $- Z \times \text{sin. vers. } 2a + \overline{R - \frac{t^3}{k^3}} \times \frac{1}{2n+1} \text{ sin. vers. } \overline{2s + \frac{1}{n}s + 2a} + \frac{S}{2} \times \frac{1}{2n} \text{ sin. vers. } \overline{2s + 2a} + \frac{S}{2} \times \frac{1}{2n+2} \text{ sin. vers. } \overline{2s + \frac{2}{n}s + 2a} + \frac{T}{2} \times \frac{1}{2n-1} \text{ sin. vers. } \overline{2s - \frac{1}{n}s + 2a} + \frac{T}{2} \times \frac{1}{2n+3} \text{ sin. vers. } \overline{2s + \frac{3}{n}s + 2a} + \frac{V}{2} \times \frac{1}{2n-2} \text{ sin. vers. } \overline{2s - \frac{2}{n}s + 2a}$

$+$

$+ \frac{V}{2} \times \frac{1}{2n+4}$ sin. vers. $\overline{2s + \frac{4}{n}s + 2a}$, &c. Eumdem hîc habet valorem quantitas Z ac in propositione præcedente. Q. E. I.

C O R O L L.

Ubi angulus PSQ est nullus, vel multiplex anguli 360°, id est, ubi planetæ versantur in conjunctione, variatio inclinationis genita generatìm est $\frac{\varphi ckn}{2t^3} Z$ $\times$ sin. vers. $\overline{2s + 2a}$ — sin. vers. $2a$ quæ, si ponatur arcus DN $= a = 0$, fit $\frac{\varphi ckn}{2t^3} Z \times$ sin. vers. $2s$.

Atque hoc est decrementum inclinationis orbis planetæ P ad orbem planetæ Q factum in qualibet serie revolutionum ad conjunctionem, initio sumpto à conjunctione factâ in nodo, vel prope ad nodum, et designante s arcum intereà à planetâ P in orbe suo descriptum.

Si inde computetur decrementum inclinationis orbis Terrestris supra planum orbitæ Veneris factum post quotcumque revolutiones Veneris ad Terram, fiet $\frac{\varphi ckn}{2t^3} Z \times$ sin. vers. $2s = 0''.3 \times$ sin. vers. $2s$, adeoque hoc decrementum, ubi maximum evadit, non superat $0''.6$, ac proinde in omni casu negligi potest.